Informational overshooting, booms, and crashes

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Abstract

This paper offers an informational explanation to stock markets’ booms and crashes. This explanation builds on the idea of ‘informational overshooting’: if market fundamentals change for an unknown period of time, prices experience a boom, which ends in a crash, due to informational dynamics. The paper then shows that ‘informational overshooting’ occurs when the market expands to a new capacity, which is unknown until it is reached. The paper presents two examples of such expansions, one due to increased productivity and the other due to entry of new investors to the stock market. One implication is that financial liberalizations tend to be followed by booms and crashes. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

This paper presents an explanation to stock markets’ booms and crashes, which is based on informational dynamics. Episodes of booms and crashes have occurred in many stock markets since the famous South Sea Bubble.¹ The US stock market has experienced two such episodes during this century: the boom and crash of 1929, and the boom and crash of 1987. The explanation that this paper offers to this phenomenon builds on the possibility of ‘informational

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¹ See Garber (1990) for description of two early famous episodes of booms and crashes.
overshooting’, i.e., it shows that if fundamentals are changing until some future time, which is unknown in advance, prices go through a boom and a crash. The paper then shows that such ‘informational overshooting’ occurs when the market goes through an expansion and the size of the expansion is unknown until it ends. The paper presents two examples of such expansions, one is a result of rapid technical progress and the other a result of large entry of new investors to the stock market. Thus, the paper predicts that financial liberalizations, which encourage new entry, might be followed by booms and crashes.

The general idea of ‘informational overshooting’ is presented by a simple example, in which dividends of a stock are rising for an unknown period of time. The public has some prior distribution on when this process might end, which reflects current available information. As dividends continue to rise, this distribution is continuously updated toward later periods and toward higher expected returns. Hence, the stock price rises. When dividends finally stop rising, the public learns that the period of rising profits has come to an end and expectations for higher future dividends fall. Hence, the stock price falls as well.

The idea of ‘informational overshooting’ is then applied to two models of market expansion with unknown size. In the first model technical progress increases the capacity of production, but the new capacity is unknown. Firms increase their capital stock, as long as the new capacity is not reached. As a result profits increase, and stock prices, namely prices of firms, rise until full capacity is reached and then stock prices crash down. In the second model entry to the stock market is limited due to entry costs, and a reduction of such costs encourages a new group of investors, who have not been in the market before, to enter. The size of this new group of entrants is unknown a priori. Their entry leads to capital accumulation, and hence to rising profits. As a result, stock prices rise as long as new investors enter, but when this group is exhausted, prices crash down.

The phenomenon of booms and crashes has long puzzled both participants and observers of stock markets. The most common explanation to such episodes has been to regard them as outbursts of irrational speculation – ‘manias’, ‘panics’, etc.\(^2\) A more recent approach interprets such episodes as rational bubbles or as sunspots, namely as alternating realizations of multiple equilibria.\(^3\) This paper follows a different approach, by assuming that speculators are rational and that equilibrium is unique, and by explaining the boom and crash

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\(^2\) Examples of this view are Galbraith (1954), Malkiel (1985) and Kindleberger (1978). A recent theoretical contribution to this view is De Long et al. (1990). Garber (1989, 1990) cast some doubts on this approach.

\(^3\) This idea originated in Blanchard (1979). For a survey of bubbles and sunspots in rational expectations, see Blanchard and Fischer (1989), (Chapter 5).
by informational dynamics. Of course, a rational model of booms and crashes should not rule out the important role played by irrational speculation and mass psychology during such episodes. It merely suggests that there may be an underlying economic process.

The paper has been motivated by two stylized facts which characterize many episodes of booms and crashes. The first is that after the crash, stock prices are still higher than at the beginning of the episode. This fact, that stock prices rise overall, indicates that profitability increases during the episode. This observation does not conform neither with irrational speculation, nor with arbitrary realization of one equilibrium out of many. Instead, it indicates that stock prices overshoot. The second stylized fact which has motivated this paper is that many booms and crashes have followed financial liberalizations. This indeed fits one of the predictions of this paper, i.e., a financial liberalization reduces entry costs, induces entry of new investors and thus, generates a boom and a crash.

Informational overshooting occurs when investors search for missing market information. Hence, this paper is related to works of Zeira (1987, 1994), Rob (1991) and Caplin and Leahy (1993), who study investment in face of missing information. While these authors examine investment behavior, this paper extends the analysis of missing information to analyze asset prices and the effect of aggregate learning on their dynamics.

The paper is organized as follows. Section 2 presents the basic idea of informational overshooting. Section 3 presents a model of capital accumulation with an unknown bounded capacity. Section 4 presents a model of entry to the stock market of a new group of investors with unknown size. Both models generate a boom and a crash. Section 5 compares some historical episodes with the predictions of the models. Section 6 concludes the paper, and mathematical proofs are given in Appendix A.

2. Informational overshooting

In this section we present the idea of ’informational overshooting’ by a simple partial equilibrium model of a market for a single stock. Consider a stock which pays a dividend $D(t)$ in time $t$, where time is assumed to be continuous. The stock is traded in a perfectly competitive market and its price is $P(t)$. Individuals who

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4 Two recent papers, by Gennotte and Leland (1990) and by Romer (1993), share the motivation of this paper, i.e. to present a rational model of booms and crashes, but they offer very different explanations from this one.

5 This has been observed in 1987 and in 1929 as well, if we consider the immediate crash, as can be seen in White (1990a). The same pattern appears in the booms and crashes in Chile (see Section 5) and Japan and in other episodes as well.
trade in the stock are risk-neutral and have rational expectations, namely they efficiently use all available information to form their expectations. In addition to stocks there is a riskless bond with a riskless interest rate \( r > 0 \). The equilibrium price of the stock is therefore equal to the expected discounted sum of dividends

\[
P(t) = E_t \left[ \int_t^\infty D(s)e^{-rs} \, ds \right],
\]

where \( E_t \) are expectations based on information in period \( t \). Consider now a situation where dividends are growing over time, but this growth will end in some future unknown date, after which dividends will stay unchanged. Formally, assume that dividends follow the time path:

\[
D(t) = D_0 e^{at}, \quad \text{for } t < T,
\]

\[
D(t) = D_0 e^{aT}, \quad \text{for } t \geq T,
\]

where \( a > 0 \). The end of dividends growth, \( T \), is unknown in advance and becomes known only when reached. The public has some prior information on \( T \), which is summarized by a prior distribution function \( F \). This distribution function is assumed to be continuous, with a density function \( f \).

The only missing variable in the market is therefore \( T \) and the public continuously learns about it. Since expectations are rational the public updates the distribution of \( T \) in the following Bayesian way. If in period \( t \) dividends are still increasing and \( T \) is not yet reached, the public learns that \( T \) exceeds \( t \) and the updated distribution function \( F_t \) is

\[
F_t(T) = \frac{F(T) - F(t)}{1 - F(t)}.
\]

Once dividends stop increasing and \( T \) is reached, it becomes fully known and the distribution shrinks to a single point. Fig. 1 presents how the distribution changes over time for a simple example of uniform distribution. As long as dividends are increasing, the distribution is squeezed to the right, from density \( f_1 \) to density \( f_2 \) for example. Once \( T \) is reached, the distribution shifts to the left, from density \( f_2 \) prior to the realization to a point distribution at \( t_2 \) following the realization.

We now turn to analyze price dynamics. Let \( P_\infty(T) \) denote the long-run price of the stock, after dividends stop increasing at \( T \):

\[
P_\infty(T) = \frac{D_0 e^{aT}}{r}.
\]
hence $p(t)$ is deterministic. This price is the expectation of the discounted sum of dividends for all possible realizations of $T$:

$$p(t) = \int_t^\infty \frac{f(T)}{1 - F(t)} \left[ \int_t^T (D_0 e^{as}) e^{-r(s-\tau)} ds + \int_t^T (D_0 e^{aT}) e^{-r(s-\tau)} ds \right] dT.$$  (5)

From Eqs. (5) and (4) we get

$$p(t) = \int_t^\infty \frac{f(T)}{1 - F(t)} \left[ \int_t^T rP_L(s) e^{-r(s-\tau)} ds + \int_T^\infty rP_L(T) e^{-r(s-\tau)} ds \right] dT.$$  (6)

Eq. (6) describes the price before $T$ is revealed. From this equation we can also calculate, by simple derivation, the change in this price over time and get

$$\dot{p}(t) = \left[ r + \frac{f(t)}{1 - F(t)} \right] [p(t) - P_L(t)].$$  (7)

We next use Eqs. (6) and (7) to derive the following proposition, which establishes the boom and the crash.

**Proposition 1.** The stock price $p(t)$ satisfies:

1. The price exceeds the long-run price: $p(t) > P_L(t)$ for all $t$, except for $F(t) = 1$.
2. The price is increasing: $\dot{p}(t) > 0$. 

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**Fig. 1.**
Proof. Note, first that $P_L(s) > P_L(t)$ for all $s > t$, since dividends are increasing. Hence, substituting $P_L(t)$ in Eq. (6) leads to

$$p(t) > \int_t^\infty \frac{f(T)}{1 - F(t)} \left[ rP_L(t) \right] \int_t^\infty e^{-r(s-t)} ds \, dT = P_L(t).$$

Hence the price is higher than the long-run price. From Eq. (7) it follows immediately that $\dot{p}(t) > 0$. □

Proposition 1 shows that the price is going through a boom that ends in a crash, as described in Fig. 2. The price rises as long as dividends are increasing and when they stop increasing in time $T$, the stock price crashes from $p(T)$ to $P_L(T)$. We say that during the boom part of the episode the price overshoots, since it reaches levels that are higher than the long-run level. This is informational overshooting, since it is a result of the missing information on when dividends stop increasing.

The intuitive explanation for the boom and the crash lies in the evolution of information during the episode. This can be seen in the dynamics of the

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6 Note that there is a possibility of no crash, or of a small crash, if the distribution is exhausted and $F(t)$ is close to 1. But this is a low probability event.
distribution function, as described in Fig. 1. As long as dividends increase and \( T \) is not reached, the duration distribution is pushed to the right and as a result the public expects dividends to be higher on average. This is why the price rises during the boom. When dividends finally stop increasing, the public updates the distribution downwards, from \( f_2 \) to a point distribution concentrated in \( t_2 \), i.e., the distribution shrinks to the left and this sends the price crashing down.

Note, that the large fluctuations in stock prices that our model predicts appear when dividends change very little. The crash occurs when dividends do not change at all but rather stop increasing. The large fluctuations reflect large changes in the future expected path of dividends. In this respect there is similarity between this paper and Barsky and De Long (1990), who claim that the crash of 1929 has been a result of a change in the path of future dividends. What our model adds is a description how such a change in expectations can be rational, under reasonable assumptions.

3. Productivity and booms and crashes

In this section we present a general equilibrium model of capital accumulation and analyze the reaction of the stock market to a large increase in productivity. As a result the economy goes through a period of expansion, but the final size of this expansion is unknown until it is reached. We show that such an expansion gives rise to informational overshooting and to a boom and a crash.

Consider a continuous time model of a small open economy in a world with one physical good \( Y \), which is produced by capital. A firm with a capital stock of \( k \) units produces \( Rk \) units of \( Y \). Aggregate productive capital in the economy is bounded by a level \( X \), i.e.,

\[
Y(t) = \min\{RK(t), RX\},
\]

where \( Y(t) \) is aggregate output at time \( t \) and \( K(t) \) is aggregate capital in all firms at time \( t \). The aggregate level \( X \), at which marginal productivity of capital drops to zero, is not modeled here explicitly, but can be an outcome of bounded technology or of bounded productivity of other factors of production. The level \( X \) is unknown until it is reached. All the prior information on \( X \) is summarized in a prior distribution function \( G \) with a density function \( g \).

We now turn to describe firms’ investment. Each firm can increase its stock of capital at a rate bounded by \( a \), where \( a > 0 \). This bound is due to adjustment costs, which can be technical, entrepreneurial, or managerial. We assume for simplicity that capital does not depreciate. We further assume that investment is financed from retained earnings only and that the outstanding number of stocks
of each firm stays constant when the firm expands.\footnote{This is an extreme assumption for the sake of simplicity and tractability. What we need here is an assumption that not all investment is financed by stocks, so that the amount of capital per stock rises over time and so are dividends. It is a well-documented fact that most investments are not financed by stock issue. See Mayer (1988).} Since production by capital and investment have constant returns to scale, we can analyze this sector as a single aggregate competitive firm. Let us denote the number of stocks in the economy by $N$.

There is a continuum of size 1 of consumers in the economy, who derive the following utility from consumption:

$$U_t = E_t \left( \int_t^{\infty} c_s e^{-\rho(s-t)} ds \right),$$

(9)

where $c_s$ is consumption in time $s$. Namely, consumers are risk neutral. As assumed above the economy is open and small with full capital mobility. The world interest rate is $r$, where it is assumed for the sake of simplicity that the interest rate is equal to the individual discount rate, i.e., $r = \rho$. We also assume that markets are perfectly competitive and that expectations are rational. Finally, assume that $R > r$.

As can already be seen from the description of the model, capital accumulation in the economy is bounded. Firms invest as long as this bound is not reached, and dividends increase as capital is accumulated. This process stops only when full capacity is reached and then dividends stop rising too. Hence, this model has similar properties to the example presented in Section 2 and prices follow a boom and a crash as well.

We begin the analysis of equilibrium with the investment decision of firms. A firm that increases its capital by $\Delta k$ in time $t$ increases its value by

$$\Delta k \frac{P(t)N}{K(t)},$$

where $P(t)$ is the price of stocks at $t$, but bears a cost $\Delta k$. Hence, if

$$q(t) = \frac{P(t)N}{K(t)} \geq 1$$

(10)

investment is maximized, i.e., capital grows at a rate $a$. Note, that $q(t)$ is the value of one unit of capital and is equal to the famous Tobin’s $q$.

If the economy reaches a steady state at a level of capital $K$, the stock price is equal to $RK/rN$. Note, that this price satisfies Eq. (10) and hence capital grows as long as $K < X$. From this we draw two conclusions. First, the steady state is
reached only at $X$, and the long-run price $P(L)$ is equal to

$$P_L(X) = \frac{RX}{rN}. \quad (11)$$

Second, investment is positive and capital grows at the maximum rate as long as $X$ is not reached. Hence,

$$K(t) = K_0e^{at}, \quad (12)$$

as long as $K(t) < X$.

We now turn to calculate the stock price before $X$ is reached, which we denote by $p(t)$. Using Eq. (12) we can derive from the distribution $G$ of $X$ the time distribution of reaching the bound, which we denote by $F$. Namely, $F(T)$ is the probability that $X$ is reached before $T$:

$$F(T) = G(K_0e^{aT}). \quad (13)$$

Denote the density function of this distribution by $f$. The price $p(t)$ is the expected discounted sum of dividends:

$$p(t) = \int_0^T f(T) [\int_0^T \frac{(R - a)K(s)}{N} e^{-r(s-t)} ds + \int_T^\infty \frac{RK(T)}{N} e^{-r(s-t)} ds] dT. \quad (14)$$

From Eq. (14) we derive the following differential equation

$$\dot{p}(t) = \left[ r + \frac{f(t)}{1 - F(t)} \right] [p(t) - P_L[K(t)]] + \frac{aK(t)}{N}. \quad (15)$$

We can now prove that stock prices go through a boom and a crash in this example as well.

**Proposition 2.** Stock prices satisfy:

1. The price exceeds the long-run price: $p(t) > P_L[K(t)]$, for all $t$, except for $F(t) = 1$.
2. Stock prices are rising: $\dot{p}(t) > 0$.

**Proof:** See Appendix A.

In order to complete the discussion of equilibrium in this model note that Proposition 2 implies that investment is indeed positive as long as $X$ is not reached. To see this note that.

$$\frac{p(t)N}{K(t)} > \frac{P_L[K(t)]N}{K(t)} = \frac{R}{r} > 1.$$

Hence, the equilibrium is consistent.
We can now turn to the economic implications of this model. We have shown that if the economy expands through accumulating capital, but the expansion is bounded by an unknown productivity bound, the stock market follows a boom that ends in a crash when the bound is reached. Hence, price dynamics in this model follow the pattern shown in Fig. 2 as well.

The model presented in this section serves two goals. First, it embeds the idea of informational overshooting of Section 2 in a general equilibrium model. Second, it describes a possible situation in which a boom and a crash might occur. In a period of rapid technical progress, productivity increases and induces an expansion of capital and output. The size of the expansion is unknown a-priori, especially if it is large, namely if technical progress is substantial. In such a case the stock market will experience a boom and a crash. Namely, economies that go through a period of rapid growth are more likely to experience a boom and a crash than economies who grow at moderate rates.

4. Entry of new investors and booms and crashes

An expansion of the economy can be triggered not only by an increase in productivity, as described in Section 3, but also by an increase in demand for capital. This is true in a closed economy, or in an economy with limited capital mobility, where demand for capital can bound capital accumulation. In this section we present an example of such increase in demand, due to an entry of a new group of investors with an unknown size to the stock market. We show that such entry generates a boom and a crash.

Consider a closed economy with one physical good $Y$, used for consumption and investment. For simplicity assume that the physical good is not storable, and fully depreciates if not invested. Assume that the economy consists of overlapping generations of individuals who live $L$ units of time each. All generations are of the same size and the flow of individuals born (and die) in time $t$ is 1. An individual born in $t$ has the following utility from consumption:

$$U_t = E_t \left[ \int_t^{t+L} c_s e^{-r(s-t)} ds \right],$$

where $r > 0$. Individuals receive an endowment when born of three possible sizes: $w_j, j = 0, 1, 2$, where $w_0 > w_1 > w_2$. The probability of receiving $w_j$ is $n_j$, where: $n_0 + n_1 + n_2 = 1$. These probabilities, or the sizes of the three endowment groups, are not known a-priori and can be revealed by the market only.

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8 This assumption is added to ensure that individuals purchase stocks only once, in order to make the model tractable.
The production in this economy is carried out by firms and by use of capital only as in Section 3. Output is described by the following production function:

\[ Y(t) = RK(t), \]  

(17)

and there is no productivity bound on capital. Investment and its finance are exactly as in Section 3. In order to derive differential entry to the stock market we assume that entry to the market is costly and an individual who purchases stocks has to bear an upfront cost of size \( z \). We assume that this cost cannot be shared and stocks cannot be purchased short.

An individual born at time \( t \) with endowment \( w_j \) can either consume all the endowment immediately, or invest it in stocks. The individual purchases stocks if

\[ P(t) \leq \left( 1 - \frac{z}{w_j} \right) \mathbb{E} \left[ \int_0^{\tau + L} D(s)e^{-r(s-t)}ds + e^{-rL}P(t + L) \right]. \]

(18)

One result that emerges from this condition is that once stocks are bought, they are held until end of life. Hence, trade in stocks is between the very old and the very young in each time. A second result is that individuals tend to buy stocks if their endowment \( w_j \) is high. Let us assume that initially entry costs \( z \) are relatively high, so that only those with endowment \( w_0 \) buy stocks. This reveals the share of such individuals \( n_0 \). The initial steady-state is therefore a price:

\[ P_0 = \frac{w_0 n_0 L}{N}, \]

(19)

and quantity of capital:

\[ K_0 = w_0 n_0 L, \]

(20)

so that the long-run price of a unit of capital is 1. While \( n_0 \) is now known, \( n_1 \) and \( n_2 \) are yet unknown. Let \( F \) be the a-priori distribution of \( n_1 \), between 0 and 1 - \( n_1 \).

Assume now that entry costs \( z \) are reduced in time 0, due to technical progress in the stock market, or due to some financial liberalization. As a result, individuals born with endowment \( w_1 \) begin to purchase stocks, stock prices begin to rise and hence \( q \) exceeds 1 and capital begins to expand if the following condition holds:

\[ \left( 1 - \frac{z}{w_1} \right) \left[ \frac{R}{r} - e^{-rL} \left( \frac{R}{r} - 1 \right) \right] > 1. \]

(21)

We can now describe the dynamics of the economy following the reduction of entry costs. Denote by \( p(t) \) be the price of stocks as long as \( n_1 \) is not revealed. If
$n_1$ is realized at time $T$, then,

$$p(T) = \frac{w_0n_0 + w_1n_1L}{N} = P_1,$$

(22)

where $P_1$ is the new long-run price. In this case capital continues to be accumulated until time $S(P_1)$, which is given by

$$K_0e^{\alpha S(P_1)} = (w_0n_0 + w_1n_1)L = P_1N.$$

(23)

Denote by $P(t, P_1)$ the price of stocks at time $t$, if $n_1$ is already known and the long-run price is $P_1$. If this price is below $P_1$ then condition (18) holds with equality for $w_1$. We assume that life horizon $L$ is long enough so that by the end of life the long-run price prevails. Hence, if price is below $P_1$, then $P(t, P_1) = V(t, P_1)$, where:

$$V(t, P_1) = \left(1 - \frac{z}{w_1}\right)\left[\int_t^{S(P_1)} \frac{(R - \alpha)K_0e^{\alpha s}}{N}e^{-r(s-t)}ds + \int_t^{t+L} RP_1e^{-r(s-t)}ds + e^{-rL}P_1\right].$$

(25)

The price after $n_1$ is revealed is, therefore,

$$P(t, P_1) = \min\{V(t, P_1), P_1\}.$$  

(26)

Lemma 1. If life horizon $L$ is long enough and if $(1 - z/w_1)(R/r - 1) < 1$, the functions $V$ and $P$ satisfy:

1. The function $V(t, P_1)$ is increasing in $t$ and in $P_1$.
2. The function $P(t, P_1)$ is increasing in $P_1$ and non-decreasing in $t$.
3. The partial derivative of $P$ with respect to $P_1$ is bounded by 1.

Proof. See Appendix A.

From here on we assume that indeed life horizon $L$ is long enough and that the condition $(1 - z/w_1)(R/r - 1) < 1$ holds, so that the results of Lemma 1 always hold.

We next examine the price before the realization of $n_1$. Note, that the prior distribution $F$ of $n_1$ can be translated to a distribution $G$ of the long-run price $P_1$, by relationship (22). If by time $t$ the long-run price has not been reached yet and demand for stocks keeps increasing, the distribution of prices is updated in the following way:

$$G_t(P_1) = \frac{G(P_1) - G[p(t)]}{1 - G[p(t)]}.$$  

(27)
Hence, the price of stocks before \( n_1 \) is revealed, is equal to

\[
p(t) = \int_{p(t)}^{P} \frac{g(P_1)}{P(t) - G[p(t)]} dP_1,
\]

(28)

where \( g \) is the density function of \( G \), and \( \bar{P} = N^{-1}(w_0n_0 + w_1 - w_1n_0)L \) is the maximum long-run price. Note, that existence and uniqueness of the price \( p(t) \) are not trivial and are dealt with in the next proposition.

**Proposition 3.** An equilibrium price \( p(t) \) always exists. The equilibrium price is unique if the distribution \( G \) is sufficiently smooth.

**Proof.** See Appendix A.

From now on we assume that the prior distribution is sufficiently smooth so that the equilibrium price \( p(t) \) is unique. The following proposition describes the properties of equilibrium prices.

**Proposition 4.** Stock prices satisfy:

1. \( p(t) > P[t, p(t)] \), as long as \( p(t) < \bar{P} \).
2. \( p \) is increasing in \( t \).

**Proof.** See Appendix A.

We can now fully summarize the dynamics of the stock market and of capital accumulation in this case, as described in Fig. 3. The reduction of entry costs begins a process of entry to the stock market of the middle class, increased demand for stocks and as a result increase in stock prices. Hence, capital is accumulated as well. The price keeps increasing as long as there is potential demand for stocks. When at time \( T \) this demand is exhausted and the price stops rising, it falls from \( p(T) = P_1 \) to \( P(T, P_1) \), which is lower according to Proposition 3. After this crash capital continues to accumulate until time \( S(P_1) \) and stock prices keep rising until they return to the long-run price \( P_1 \). Thus stock prices follow a boom, a crash and then a gradual rise back to the high levels of the boom. Hence, entry of a new group of investors to the market, of an a-priori unknown size, can also generate a boom and a crash.

Another implication of this section is that a financial liberalization can also lead to a stock market boom and crash. Such a liberalization removes barriers to entry to the stock market and thus can be viewed as reducing entry costs. As a result new groups of investors, individuals or institutions or both, enter the market. If the size of this new entry is not a priori known, the stock market goes through a boom that ends in a crash, when the size of the new group of entrants
becomes known and when their entry slows down and stops. Hence, the model has a clear empirical implication, that financial liberalizations tend to generate booms and crashes.\(^9\)

5. The booms and crashes of 1929 and 1987

In this section we look at some historical episodes of booms and crashes and compare them with the implications of our theory. We focus mainly on the two famous US episodes, the boom in the 1920s, which ended in the crash of October 1929, and the boom in the 1980s, which ended in the crash of October 1987. Interestingly, those two episodes had quite similar price dynamics, as observed by White (1990b). Note, that both episodes are still a subject of intense research by economists and to a large extent are still a puzzle.\(^{10}\)

Both the 20s and the 80s have been expansionary periods in the US. Between the years 1921 and 1929 GDP has grown at an average rate of 4.8% a year and output per capita has grown at an average rate of 3.3%. These are very high

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\(^9\)Note, that in the rare case that the price reaches the upper bound \(P\), there is no crash.

\(^{10}\)Recent examples for the intense ongoing research on the boom and crash of 1929 are De Long and Shleifer (1990), Rappoport and White (1991) and White (1990a,b).
rates of growth relative to other periods in American history. The 80s have also been a period of expansion after a short recession in 1981-1982. Between the years 1982 and 1989 GDP has grown at an average rate of 3.7% in what was considered to be the longest post-war expansion.\footnote{Data are from Maddison (1995).}

The two American episodes are also characterized by large entries of new investors to the stock market. There is few data on shareownership from the 20s, but we can learn from the little we have on large entry of investors to the market. The Senate Committee on Banking and Currency (1934) reports that in 1929 there were 1,548,707 customers in all US exchanges, which is 1.3% of total population at the time. The same report finds, that between January and the end of July in 1929 margin traders increased from 317,501 to 369,093, namely by more than 16%. Instead of direct data on customers, we have data on securities firms and their offices. The number of offices of member organizations in the NYSE has grown from 1,225 in 1919 to 2,141 in 1930.\footnote{Data are from NYSE (1989).} This increase in broker services indicates that there has been large entry of new investors into the market during the twenties.

There is much more available data on shareownership in the 80s, from surveys conducted by the NYSE in the years 1980, 1981, 1983, 1985 and 1990. These surveys show a massive entry of new investors to the stock market during that period. The number of shareowners of public corporations and mutual funds has increased from 30 million in 1980 to 47 million in 1985 and to 51.4 million in 1990. Not only the number of shareowners increased, but their percentage in the total population as well. The share of shareowners in the population increased from 13.5% in 1980 to 20.1% in 1985 and to 21.1% in 1990.\footnote{The data are from NYSE (1991).} Note, that entry to the stock market has been even greater than what is indicated by these figures, due to entry of many foreign investors in the 80s, as part of the huge capital inflow in this period.\footnote{See Finnerty and Czajkowski (1992).} Additional information on stock market activity can be obtained from data on sale offices of member organizations in the NYSE and on employment in the securities industry. The number of NYSE member firms’ offices increased from 4,421 in 1980 to 6,969 in 1987, and fell after the crash to 6,795 in 1988. Employment in the securities industry increased from 140 thousands in 1980 to 260 thousands in 1987, and fell after the crash to 239 in 1988.\footnote{The data are from SIA (1990).} These figures indicate that between 1980 and 1987 the market experienced a sharp increase in activity, due partly to a large entry of new investors to the market.
Hence, both episodes have been accompanied by a real expansion and by entry of many new investors to the market. We next examine potential triggers for the two episodes. The paper suggests two possible triggers to booms and crashes, namely rapid technical progress and reduction of entry costs to the market. It seems that in the 20s both triggers played their part. These have been years of rapid technical progress in American industry, construction, communication and agriculture. These have also been years of rapid capital accumulation and in some sense of overaccumulation. In 1929 the capital-output ratio reached a high level of 4.3, and it later declined to 2.7 in 1945 and to 3.2 in 1960.\(^{16}\) But the 20s were also years of reduction of entry costs to the stock market, due to better access to information, as a result of technical progress in communication. The Senate Committee on Banking and Currency (1934) writes: ‘In former years transactions in securities were carried on by a relatively small portion of the American people. During the last decade, however, due largely to development of the means of communication – the expanding network of telephone, telegraph, ticker, radio, and newspaper facilities – the entire Nation has become acutely sensitive to the activities on securities exchanges.’

The 80s seem to be a classical case of a reduction of entry costs to the stock market. There has been a substantial deregulation in US financial markets in the late 70s and early 80s. This includes the deregulation of the S and L industry, the deregulation of Pension Funds, and other financial sectors. This deregulation reduced entry costs and enabled entry of new investors, both private and institutional, into the market. Also, Smidt (1990) documents a dramatic increase in turnover rate during the 80s, which he attributes to a reduction in transaction costs. Hence, the boom and crash of the 80s followed a financial liberalization, as predicted by the entry model in Section 4.

A close look at other recent episodes of booms and crashes shows that, like the American episode of the 80s, they too have been preceded by financial liberalizations. One such example has been Chile during the 70s and early 80s. Financial liberalization in Chile began in 1974 and from 1975 the Chilean stock market went through a remarkable boom.\(^{17}\) From a level of 100 by the end of 1975, the real value of stock prices rose to 240 by the end of 1977, to 544 by the end of 1979 and to a peak of 1020 in June 1980. Since then stock prices fell drastically down to a level of 400 at the beginning of 1983.

Other recent examples of booms and crashes that have been preceded by financial liberalizations are Indonesia, Japan and Israel. Indonesia liberalized financial markets in the end of 1988 and after that the stock market boomed and then crashed in February 1990. The Japanese stock market boom in the 80s and


\(^{17}\) A detailed description of the financial liberalization in Chile appears in Hanna (1987).
its crash in the early 90s followed a financial deregulation. Another recent example is Israel, which liberalized financial markets in the late 80s and early 90s, among other things allowing provident funds to invest in stocks. In addition the economy experienced a rapid expansion as a result of a large immigration wave in 1990 and 1991 from the former Soviet Union. Indeed, in the early 90s the Israeli stock market experienced a dramatic boom, which ended in a crash in May 1994.

6. Summary

This paper offers a rational explanation to the phenomenon of stock market booms and crashes. It claims that if fundamentals change for an unknown period of time stock prices overshoot, and we call such a reaction ‘informational overshooting’. The paper then relates episodes of booms and crashes to periods of economic expansion. It shows that if an economy experiences an expansion of unknown size, its stock market goes through a boom and a crash. Note, that missing information on the scale of expansion is inherent to such expansions. An expansion is like exploration of new unknown territory. We know the market and the economy before the change, but the expansion carries us to the unknown, both in quality and in quantity. This paper analyzes the asset prices’ implications of such missing information.

There are three important points to note here. The first is that the paper in effect suggests that ‘fundamentals’ should include more economic variables than just dividends and discount rates. It should include all relevant information held by the public, whether on actual activity or on potential future activity. Note that this information changes both through news and through learning, and it therefore changes in highly nonlinear ways, as shown in the paper.

The second point is about symmetry. Informational undershooting can occur when dividends are decreasing. But although such a scenario is theoretically feasible, it is much less likely. Note, that according to our examples, informational overshooting occurs during expansions and the likelihood of expansions is greater than that of contractions in growing economies. The empirical evidence on stock prices indeed justifies our focus on informational overshooting, since stock price crashes are much more common than stock price hikes, as shown by Friedman and Laibson (1989).

The third point is that although this paper suggests a rational explanation to booms and crashes, we should not rule out the importance of irrational speculation during such episodes. Mass psychology is a force that intensifies both the boom and the crash. This paper only claims, that it is possible to identify, beneath the manias and the panics, some underlying economic process, which generates the boom and the crash.
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Appendix A.

Proof of Proposition 2
Let us denote the present discounted value of a stock if production bound is met at $T$ by $V(T)$:

$$V(T) = \int_t^T \frac{(R-a)K(s)}{N}e^{-r(s-1)}ds + \int_T^\infty R\frac{K(T)}{N}e^{-r(T-1)}ds.$$ 

Note that $V(t) = PL[K(t)]$ and that

$$V'(T) = \frac{K_0e^{rt}}{N}ae^{(a-r)T}\left(\frac{R}{r} - 1\right) > 0.$$ 

Hence,

$$V(T) > P_L[k(t)]$$

for all $t$. Substitute in Eq. (14) and get,

$$p(t) = \int_t^\infty \frac{f(T)}{1-F(t)}V(T)dT > P_L[k(t)],$$

which proves the first part of the proposition. The second part follows from the first part and from Eq. (15). □

Proof of Lemma 1
Calculating the partial derivatives of $V$ yields

$$\frac{\partial V}{\partial P_1} = \left(1 - \frac{z}{w_1}\right)\left(\frac{R}{r} - 1\right) \left[e^{-r(SP_1)_t} - e^{-rL}\right] > 0,$$
if $L$ is large enough, and

$$
\frac{\partial V}{\partial t} = \left(1 - \frac{z}{w_1}\right) \int_t^{\infty} \frac{N(R - a)Ke^{as}}{N} e^{-r(s-t)} ds + r \int_s^{\infty} \frac{RK_0e^{as}}{N} e^{-r(s-t)} ds - \frac{(R - a)K_0e^{at}}{N} = 0.
$$

Hence, $V(t, P_1)$ is increasing in $P_1$ and in $t$.

If $P(t, P_1) < P_1$ we have $\partial P/\partial P_1 = \partial V/\partial P_1$. From the first derivative in this proof and from the condition in the Lemma we get: $1 > \partial V/\partial P_1 > 0$. If $P(t, P_1) = P_1$ we have $\partial P/\partial P_1 = 1$. Hence $P(t, P_1)$ is increasing in $P_1$ and the partial derivative is bounded by 1. 

**Proof of Proposition 3**

For each $t$ define a function $H_t(p)$ in the following way:

$$
H_t(p) = \int_p^P \frac{g(P_1)}{1 - G(p)} P(t, P_1) dP_1,
$$

for all $p$ in the interval

$$
\frac{K_0e^{at}}{N} \leq p \leq \bar{P}.
$$

The price $p(t)$ is a fixed point for $H_t$, or a solution to $p = H_t(p)$.

We first show existence of a solution. Note that

$$
H_t(\bar{P}) = P(t, \bar{P}) \leq \bar{P},
$$

and the inequality is strict if $t$ is small enough. Note also that due to Lemma 1 and to condition (21):

$$
H_t\left(\frac{K_0e^{at}}{N}\right) > P\left(t, \frac{K_0e^{at}}{N}\right) = \left(1 - \frac{z}{w_1}\right) \left[\frac{R}{r} - e^{-rt} \left(\frac{R}{r} - 1\right)\right] \frac{K_0e^{at}}{N} > \frac{K_0e^{at}}{N}.
$$

Hence, there exists a price $p(t)$ that satisfies: $p(t) = H_t[p(t)]$.

We next turn to analyze uniqueness of equilibrium. The slope of $H_t$ is

$$
H'_t(p) = \frac{g(p)}{1 - G(p)} [H_t(p) - P(t, p)],
$$

and the equilibrium price is unique if this slope is smaller than 1. By calculating the slope and using the bound on the partial derivative of $P$ from Lemma 1 we
get

\[ H_t = \frac{g(p)}{1 - G(p)} \int_p^\infty \frac{g(P_1)}{1 - G(p)} [P(t, P_1) - P(t, p)] dP_1 \leq \frac{g(p)}{1 - G(p)} \times \int_p^\infty \frac{g(P_1)}{1 - G(p)} (P_1 - p) dP_1. \]

Note that if the distribution is uniform the right hand side in the above inequality is equal to 1/2. Hence, if the density function does not have too large variations, we get \( H_t'(p) < 1. \)

**Proof of Proposition 4**

According to Lemma 1 the function \( P(t, P_1) \) is increasing in \( P_1 \). Hence, \( P(t, P_1) > P(t, p(t)) \) for all \( P_1 > p(t) \). Hence, we get:

\[ p(t) = \int_{p(t)}^\infty \frac{g(P_1)}{1 - G[p(t)]} P(t, P_1) dP_1 \geq P[t, p(t)], \]

and the inequality is strict if \( p(t) < \bar{P} \). This proves the first part of the proposition.

We now turn to show that price increases before realization of \( P_1 \). Note that

\[ \hat{p}(t) \left[ 1 - \frac{g(p(t))}{1 - G(p(t))} (p(t) - P(t, p(t))) \right] = \int_{p(t)}^\infty \frac{g(P_1)}{1 - G[p(t)]} \frac{\partial P}{\partial t} dP_1. \]

From Lemma 1 it follows that the right hand side of this equation is positive. Due to our assumption that the equilibrium price \( p(t) \) is unique, the parentheses on the left hand side is positive, and hence \( \hat{p}(t) > 0. \)

**References**