Machines as Engines of Growth

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Abstract

This paper presents a model of growth through industrialization, where machines replace labor in a growing set of tasks. While substituting labor in some tasks, machines complement labor in the remaining tasks by increasing its productivity. This dual role of capital leads to a feedback between technology and wages. Higher wages induce creation and adoption of machines to replace costly labor, while these machines complement labor in the remaining tasks and raise wages further. This feedback fuels economic growth and can even lead to long-run growth. This enables the model to examine potential triggers to the industrial revolution.

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Machines as Engines of Growth

1. Introduction

This paper presents a model of global economic growth. The phenomenon of global economic growth has puzzled economists as it is a new, unprecedented and quite dramatic historical phenomenon. During the last two centuries global output per capita has grown by more than 9 fold and in the more developed countries output per capita has grown by twice as much. This rapid growth began with the industrial revolution, somewhere around 1820, according to Maddison (1995, 2001). This paper claims that modern economic growth has been made possible by a new type of technology, machines that can replace human workers in a growing set of tasks The paper shows that the introduction of such machines can contribute significantly to understanding modern global economic growth.

This engine of growth emerges as a result of the dual effect of capital on labor in such an economy. First, capital replaces labor in more and more tasks, or rather in the production of more and more intermediate goods. Second, capital complements labor that concentrates in the production of the remaining intermediate goods, by enabling labor to operate with more machines, to perform less tasks, and thus to produce more. For example, consider builders who begin to use a crane that can lift materials up the building, instead of carrying them manually. This enables the same workers to finish the building earlier, namely to increase production.

This dual effect of capital on labor, of supplementing some tasks and complementing others, creates a feedback effect between technical progress and wages.

On one hand, producers demand the new technology that replaces labor by machines only if wages are sufficiently high. On the other hand, the use of more machines complements labor in the remaining tasks and thus raises wages. This feedback between growth and wages can therefore explain how growth continues over time.

Recently there has been renewed interest in the role of wages as incentive to technical change, as more empirical evidence points in this direction.[1] One of the most important contributions to this accumulation of empirical support is the new book by Allen (2009) on the role of factor prices in the takeoff of the industrial revolution in England at the time. This carefully documented book shows that the industrial revolution was preceded by a significant rise in wages in England at the 18th century and also by a decline in coal prices. It therefore claims that these two developments were crucial for the takeoff of the industrial revolution.

This paper presents a theoretical model that analyzes this relationship between wages and technology through industrialization. Since standard modeling of innovation does not fit such relationship, what is required here is a theory of what Acemoglu (2009) calls 'labor cost induced innovations,' which is supplied by the model of machines that replace workers. But this theory does more than identifying the engine of modern economic growth. It also helps in understanding which factors can stop or ignite global economic growth. According to this model growth is negatively related to the cost of machinery, and it is positively related to the overall productivity of the economy, which is fixed overtime, and reflects geography, climate, infrastructure and access to trade. Productivity affects growth through wages. The model also shows that global growth is negatively related to the cost of financial intermediation, as it increases the cost of

[1] See Acemoglu (2009).

machinery, and it is also negatively related to monopoly power, since it reduces wages. These results point at a few potential triggers to the industrial revolution. One is the invention of the steam engine, which offered a new general technology to build machines and thus reduced the overall cost of machinery. A second possible trigger could have been a rise in general productivity caused by the conquest of the Americas. A third trigger could be the development of capital markets in the 17^{th} and 18^{th} centuries. A fourth trigger could be the collapse of Feudalism, with its established monopoly rights and the rise of free labor markets.

This paper is strongly related to the two main theories of endogenous global growth, that of capital accumulation, and that of innovation. The first theory consists of the AK models of Jones and Manuelli (1990), Rebelo (1991) and others. The second theory is the R&D based endogenous growth models of Romer (1990), Segestrom, Anant, and Dinopolous (1990), Grossman and Helpman (1991), Aghion and Howitt (1992) and Jones (1995, a, b). This paper can be viewed as an attempt to bridge between these two theories, as it combines elements of both, but also differs from each significantly.

The roots of the idea that innovations substitute labor with capital can be found already in Hicks (1932). It then influenced the literature of induced biased innovation, as in Kennedy (1964), Samuelson (1965), and Habbakuk (1962).[2] This literature analyzed the bias of technical change toward capital versus labor, but did not go far enough to examine replacement of labor by capital. This has been first modeled by Champernowne (1963), who studied its effect on the aggregate production function. This idea appears

[2] See a survey in Acemoglu (2003).

again in Zeira (1998), which uses it to study output differences across countries.[3] The current paper uses this idea in a very different framework, of global economic growth, and also adds to the analysis endogenous invention of technologies.[4]

The paper is constructed as follows. Section 2 presents the model. Section 3 describes technology choice and Section 4 analyzes the dynamics of economic growth. Section 5 discusses possible triggers to the industrial revolution and to modern economic growth. Section 6 examines how the model fits the data on long-run economic growth. Section 7 compares the model with the two main theories of endogenous growth. Section 8 summarizes the paper.

2. The Basic Model

This section describes the benchmark model of growth and industrialization. Consider a closed economy, which produces one final good, which is used both for consumption and for investment. The final good is produced by a continuum of tasks, ordered on [0, 1]. Production of the final good in period t, Y_t, is described by a Cobb-Douglas production function:[5]

$$(1) \qquad \ln Y_t = \ln A + \int_0^1 \ln X_t(j)dj,$$

where $X_t(j)$ is the amount of production of task j in period t and A is a productivity parameter, which refers to the aggregate economy and is assumed to be constant over

[3] Similar models are used by Zeira (2007) and Caselli and Coleman (2006) to study skill-technology relations. Beaudry and Collard (2002) use a similar idea in analyzing employment dynamics. A related approach is that of 'appropriate technologies' of Basu and Weil (1998).
[4] Recently, a number of new papers, written independently, apply this approach to issues of economic growth. These are Zuleta (2005), Peretto and Seater (2006), Givon (2006) and Alesina and Zeira (2009). An analysis of the underlying assumption of this line of literature appears in Acemoglu (2009).
[5] Alternative production functions, like CES, yield similar results.

time. It is later shown that this productivity parameter plays an important role in the dynamics of the model.

Each task can be performed by one of two potential technologies, pre-industrial, namely manual, or industrial. Both technologies operate in fixed proportions. In the pre-industrial technology one unit of task j is performed by 1 unit of labor.[6] The industrial technology introduces a machine that can perform the same task. This machine consists of $m(j)$ units of capital and can replace one unit of labor, namely perform one unit of task j. Capital fully depreciates after one period of time, which means that time units are fairly long. It is further assumed that invention of such machines is costless, so that a machine is invented once there is demand for it.[7]

We next assume that the function m is continuous and increasing, namely the intermediate goods are ordered by increasing cost of machines:

(2) $$m'(j) > 0.$$

Namely, machines become increasingly complicated and more costly with the tasks. We next assume that the overall cost of industrialization is finite and is bounded by B:

(3) $$\int_0^1 \ln m(j)dj = \ln B.$$

As shown below the parameter B also plays an important role in the dynamics of the economy, together with productivity A.

We also assume that there are adjustment costs to net investment, namely that changing the structure of production has additional costs. These adjustment costs are standard and are described by a convex cost function with constant returns to scale in the

[6] It is possible to add structure capital as an additional input to manual production, but it does not affect the analysis significantly. Its importance is mainly empirical, as shown in Section 6 below.
[7] The case of costly innovation is analyzed in Section 6.

new and previous quantities of capital. We further adopt the standard specification of quadratic adjustment costs. Thus, the adjustment costs in period t, a_t, are described by:

$$a_t = \frac{1}{2\gamma}\frac{\left(K_{t+1} - K_t\right)^2}{K_t}. \tag{4}$$

Also assume that the capital market in each period operate after investment has been made. Hence the value of a firm in period t is $V_t(K_{t+1})$.

Finally, people in this economy are identical, they have an infinite horizon, and they come in a mass of size N. Each person supplies 1 unit of labor in each period and has the following utility from consumption:

$$U = \sum_{t=0}^{\infty} \frac{c_t}{(1+\rho)^t}. \tag{5}$$

The use of risk neutral utility is for simplification only and the results of the paper hold for other utility functions as well.

3. Technology, Industrialization and Factor Prices

The main decision facing producers in this economy is the choice of technology, namely whether to stick to the old pre-industrial technology or to industrialize. The decision depends on factor prices, since industrialization involves reduction of labor, but at the expense of purchasing machines, namely increasing the cost of capital. Let us begin the analysis of this decision by studying value maximization by producers. First, denote the set of tasks which are produced by machines in period t by F_t. Then producers maximize in period t the sum of profits and the value of their firm minus the costs of investment:

$$\max_{X_t(j), F_t, K_{t+1}} \left\{ Y_t - w_t \int_{F_t^c} X_t(j)dj - K_{t+1} - \frac{1}{2\gamma}\frac{(K_{t+1} - K_t)^2}{K_t} + V_t\left(K_{t+1}\right) \right\}. \tag{6}$$

As usual, the decision of the firm is divided into two parts. One is profit maximization, which determines the amount of each task $X_t(j)$ and the set F_t, given the quantity of capital K_t, which was determined in period t – 1. The second part is net value maximization, which determines capital for next period, K_{t+1}.

The maximization of operating profits in period t is described by:

(7) $$\max_{X_t(j),F_t} \left\{ Y_t - w_t \int_{F_t^c} X_t(j)dj \right\}, \; s.t. \;\; K_t = \int_{F_t} m(j)X_t(j)dj.$$

Note, that the set of industrialized tasks F_t is also determined in t – 1, when the amount of capital is determined, since the machines are invented then, but for analytical tractability we view K_t as determined first and then technologies. Maximization of (6) immediately implies that producers industrialize the tasks that require the machines with the lowest cost $m(j)$. Hence, the set of industrialized tasks is equal to:

$$F_t = [0, f_t].$$

The task f_t is called the frontier of industrialization, or of technology, in period t. As we show below f_t is increasing over time, so $(f_{t-1}, f_t]$ are the machines invented in t – 1. Note that these machines are invented since there is demand for them. We can therefore rewrite profit maximization (7) in the following way:

(8) $$\max_{X_t(j),f_t} \left\{ Y_t - w_t \int_{f_t}^{1} X_t(j)dj \right\}, \; s.t. \;\; K_t = \int_{0}^{f_t} m(j)X_t(j)dj.$$

Solving (8) we get the following first order conditions. For tasks that are not industrialized yet $f_t < j \le 1$ we get:

(9) $$X_t(j) = \frac{Y_t}{w_t}.$$

For industrialized tasks $0 \le j \le f_t$ we get:

(10) $$X_t(j) = \frac{Y_t}{R_t m(j)}.$$

R_t is the Lagrange multiplier of the constraint in maximization (8). Finally, the first order condition with respect to the level of industrialization f_t leads to the following condition:

(11) $$m(f_t) = \frac{w_t}{R_t}.$$

It follows from (11) that the degree of industrialization depends strongly on the wage rate in the economy. Higher wages create an incentive to invent and to use more technologies, as these enable reduction of the costly labor input.

As is clear from profit maximization the optimal frontier of industrialization, f_t, depends on the amount of capital invested in t-1, K_t and on the equilibrium wage w_t. We next turn to see how the wage is determined in equilibrium. The labor market equilibrium condition is:

(12) $$N = \int_{f_t}^{1} X_t(j)dj = (1 - f_t)\frac{Y_t}{w_t}.$$

Using the first order condition (10) we get that the amount of capital is equal to:

(13) $$K_t = \int_{0}^{f_t} m(j)\frac{Y_t}{R_t m(j)}dj = \frac{f_t Y_t}{R_t}.$$

Hence, applying the first order condition (11), the capital labor ratio satisfies:

(14) $$\frac{K_t}{N} = \frac{f_t}{1 - f_t}\frac{w_t}{R_t} = \frac{f_t}{1 - f_t}m(f_t).$$

Note that the RHS of (14) is an increasing function of the industrial frontier, and hence the industrial frontier f_t is uniquely determined. This condition therefore determines the equilibrium technology frontier, given the capital stock invested in period $t-1$. Equation (14) also implies that the two variables are positively related. Namely, as capital increases over time, technology and industrialization move with it and more tasks become industrial. Furthermore, it can be shown that:

(15) $$K \xrightarrow[f \to 1]{} \infty.$$

This means that as more and more tasks are industrialized, capital becomes infinite, as it complements labor that concentrates in an ever reduced set of tasks.

Let us next examine how the equilibrium wage rate is related to the equilibrium level of technology. Substituting the first order conditions (9) and (10) in the production function (1) leads to:

$$\ln Y_t = \ln A + \int_0^{f_t} [\ln Y_t - \ln R_t - \ln m(j)]dj + \int_{f_t}^{1} (\ln Y_t - \ln w_t)dj.$$

After some manipulation and using equation (11) we get:

(16) $$\ln w_t = \ln A + f_t \ln m(f_t) - \int_0^{f_t} \ln m(j)dj.$$

A simple derivation of (16) and a use of assumption (2) imply that wages are an increasing function of the technology frontier. Together with (11) we get that the wage rate depends positively on the level of industrialization, and in turn industrialization depends positively on wages. Hence, there is a strong positive feed-back between the two variables, w_t and f_t. Later, the paper shows that this feed-back can have significant effects on the dynamics of the economy, and it actually enables persistent growth, if some conditions are met.

As the economy goes thorough technical change and industrialization, and f_t converges to 1, the wage rate increases and output grows unboundedly. This can be seen simply from the following simple calculation of output per worker:

$$\frac{Y_t}{N} = \frac{w_t}{1 - f_t}.$$

As the nominator rises and as the denominator goes to 0, output and output per worker go to infinity. Thus, if the economy converges to full industrialization it also experiences unbounded economic growth. The reason is that as more tasks are performed by machines and less and less tasks by labor, output becomes unbounded as the capital input is unbounded as well.

4. Dynamics of Industrialization

We begin this section by analyzing the net value maximization by producers, which determines capital accumulation in the economy. Note first that the maximized operating profits depend on the amount of capital K_t in the following way. From (9) we deduce that total labor costs are equal to $(1 - f_t)Y_t$. Hence operating profits are equal to $f_t Y_t$. Using equation (13) that describes the overall amount of capital, we get that operating profits are equal to:

(17) $$f_t Y_t = R_t K_t.$$

Thus, profits are proportional to capital and the rate of return on capital is R_t. This is reasonable since production in this economy has constant returns to scale.

Note that since both production and adjustment costs have constant returns to scale, profits minus investment costs are proportional to capital as well. Hence, the market value of capital is also proportional to the quantity of capital, as shown in Hayashi (1982):

(18) $$V_t(K_{t+1}) = q_t K_{t+1}.$$

The value of a unit of capital q_t is the standard Tobin's q in this economy. Substituting (18) in (6) leads to the following maximization:

$$\max_{K_{t+1}} \left\{ R_t K_t - K_{t+1} - \frac{1}{2\gamma}\frac{(K_{t+1}-K_t)^2}{K_t} + q_t K_{t+1} \right\}.$$

Hence, the first order condition of value maximization is

(19) $$\frac{K_{t+1}}{K_t} = 1 + \gamma(q_t - 1).$$

Therefore, investment depends positively on q, as in all such models of adjustment costs.

While equation (19) describes the dynamics of capital, the dynamics of the price of capital, q_t, are determined in the capital market equilibrium. Note first that due to risk neutrality of consumers the interest rate in the economy is equal to the subjective discount rate ρ, as long as $0 < C_t < Y_t$. In this case the capital market equilibrium condition in period t can be written as equality between the gross interest rate and the gross rate of return on capital:

(20) $$1+\rho = \frac{R_{t+1}K_{t+1} + q_{t+1} - (K_{t+2}-K_{t+1})^2 / (2\gamma K_{t+1}) - K_{t+2}}{q_t K_{t+1}}$$

Some manipulation of this condition and the use of (19) for period $t + 1$ lead to:

(21) $$q_t(1+\rho) = R_{t+1} + \frac{\gamma}{2}q_{t+1}^2 + (1-\gamma)q_{t+1} - 1 + \frac{\gamma}{2}.$$

To complete the analysis note that according to (11), the rate of return on capital is equal to $R_t = w_t / m(f_t)$. According to (16) the wage w_t is a function of f_t, so R_t is a function of f_t as well, which is described as follows:

(22) $$\ln R_t = \ln w_t - \ln m(f_t) = \ln A - (1 - f_t)\ln m(f_t) - \int_0^{f_t} \ln m(j)dj.$$

A simple derivation of (22) reveals that the derivative is negative so the function is decreasing. Since the technology frontier f_t is an increasing function of capital K_t, as shown by equation (14), it follows that the rate of return is a decreasing function of the amount of capital, which can be denoted by R:

$$R_t = R(K_t).$$

Substitution in (21) leads to the following capital market equilibrium condition:

(23) $$R\left[K_t(\gamma q_t + 1 - \gamma)\right] - q_t(1+\rho) + \frac{\gamma}{2}q_{t+1}^2 + (1-\gamma)q_{t+1} - 1 + \frac{\gamma}{2} = 0.$$

Conditions (19) and (23) are the two dynamic equations of the economy that describe how capital and its price change over time. These conditions constitute a dynamic system whose solution is a saddle path, which is shown below in Figures 1 and 2. The two curves in each figure are the usual dynamic conditions of constant capital, namely $q_t = 1$, and constant prices of capital, namely $q_t = q_{t+1}$. It is easy to verify from (23) that the curve $q_t = q_{t+1}$ is downward sloping, unless q_t is very high above 1. We next distinguish between two cases. The first, which is shown in Figure 1, is when the two curves intersect at some level of capital K^*. The second case, which is shown in Figure 2, is when the two curves do not intersect and the curve $q_t = q_{t+1}$ is everywhere above the line $q_t = 1$.

In the first case capital and output are finite in the steady state. The long-run amount of capital is determined by: $R(K^*) = 1 + \rho$ and industrialization is bounded by *f**, which is determined, according to (14), by:

$$\frac{K^*}{N} = \frac{f^*}{1 - f^*} m(f^*).$$

Clearly if *K** is finite, (15) implies that $f^* < 1$, namely industrialization and technical change come to a stop at some level.

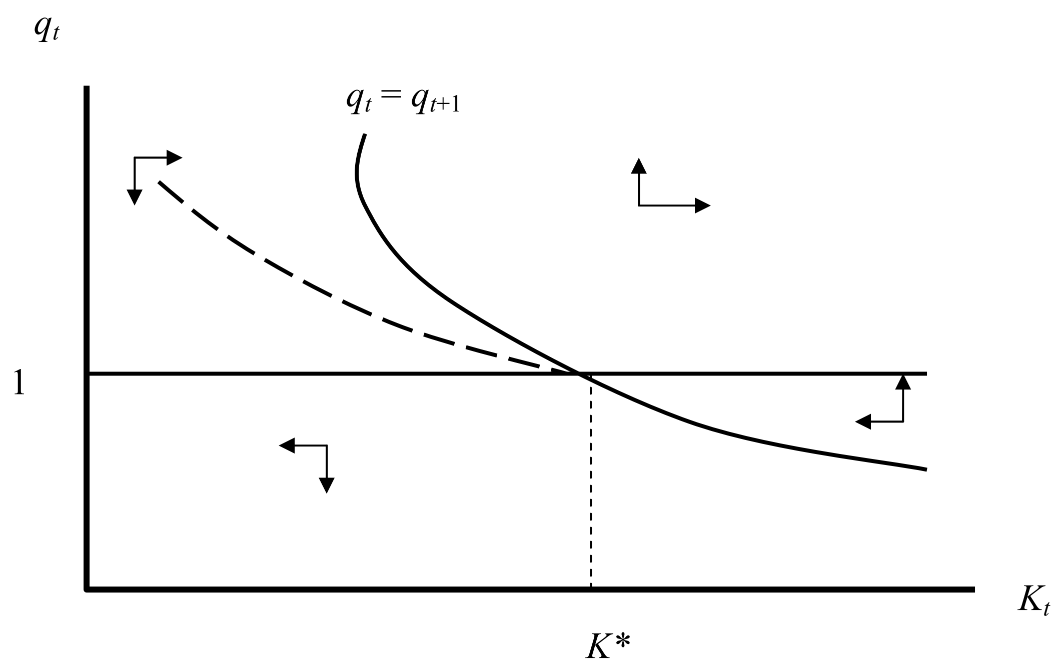


Figure 1: Bounded Growth If $A/B < 1+\rho$

The economy in the case of a finite steady state converges along the dashed curve, which is the saddle path, to the steady state at *K**. It is easy to verify that any other dynamic path leads to either negative consumption (if *q* grows too much and investment becomes greater than output), or to zero price of capital q, and both are deviations from equilibrium. To examine further the condition under which growth is bounded, note that the rate of return satisfies, according to (22):

$$\ln R = \ln A - (1-f)\ln m(f) - \int_0^f \ln m(j)dj \xrightarrow[f\to 1]{} \ln A - \ln B. \tag{24}$$

Hence, if the economy converges to an industrial level $f^* < 1$, and to a rate of return equal to $1 + \rho$, it implies that $A/B < 1+\rho$.

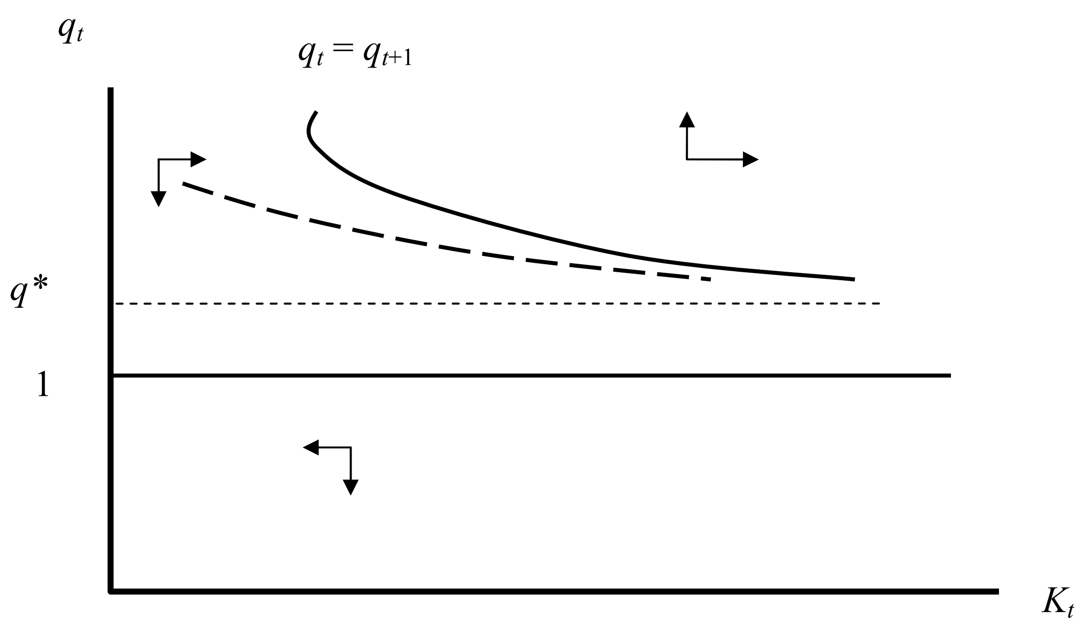


Figure 2: Unbounded Growth If $A/B \geq 1+\rho$

The second case, which is described in Figure 2, is the case where $A/B \geq 1+\rho$. In this case the price of capital is declining but never gets to 1, as it converges from above to a price q^*, $q^* \geq 1$. According to (23), this limit price is given by the following condition:

(25) $$\frac{A}{B} - q^*(\gamma + \rho) + \frac{\gamma}{2}(q^*)^2 - 1 + \frac{\gamma}{2} = 0.$$

Note that the economy grows unboundedly in this case. Condition (19) implies that as the price of capital converges to q^* the rate of growth of capital converges to:

(26) $$g = \gamma(q^* - 1) \geq 0.$$

The long-run rate of growth of capital is positive if $q^* > 1$. Note that this is also the long-run rate of growth of output and output per capita, since according to equation (17) output is equal to: $Y_t = (R_t / f_t)K_t$. As the economy grows the rate of return converges to *A/B* and the technology frontier converges to 1, so the rate of growth of output converges to the long-run rate of growth of capital *g*. Hence, the economy experiences sustainable growth in this case. This discussion can therefore be summarized in the following Proposition.

<u>Proposition 1:</u> There is a unique equilibrium growth path. If $A > B(1 + \rho)$ the economy grows forever and its rate of growth converges to $g > 0$. If $A < B(1 + \rho)$, growth and industrialization peter out at some finite level of output. If $A = B(1 + \rho)$, output grows unboundedly, but the rate of growth converges to 0.

Note first that although the model can generate long-run growth the case of bounded growth is of interest as well. This category can include many cases, from a stagnant economy without industrialization, namely when $f^* = 0$, to an economy that grows over a long period, becomes highly industrialized, and only after a long time growth peters out. In other words, the case of no long-run growth can fit even our actual world under some parameters. Another interesting aspect of Proposition 1 is the strong dependence of the growth path on the three parameters, the overall productivity of the economy A, the overall cost of machinery B, and the required discount rate ρ. A one-time shock to each of these parameters can lead to increased growth over a long period of time and even to permanent growth, as discussed in the next section. To gain a better understanding of equilibrium in this economy, the next proposition analyzes the aggregate production function and the optimality of the competitive equilibrium.

Proposition 2: Let $F(K,L)$ be the maximum amount of output produced by L workers and K capital. Then the equilibrium amount of output is described by this aggregate production function $Y_t = F(K_t, N)$. Also, the marginal productivities of labor and of capital of F are equal to w_t and R_t, respectively. Furthermore, the equilibrium is optimal.

Proof: In the Appendix.

5. What Can Trigger Economic Growth?

As Maddison (1995, 2001) shows, the rapid economic growth is a fairly recent historical phenomenon that started sometime in the beginning of the 19th century and has been going steadily ever since. Economic growth is also inherently related to the process of industrialization. What could have triggered this process that changed society and economy so dramatically over the last two centuries? We show in this section that our model can offer a number of potential answers to this question. These possible answers can be better understood by Figure 3, which plots the R curve, the value of the gross rate of profit R along the path of industrialization, as implied by equation (22). Figure 3 also plots the horizontal curve of $1 + \rho$. There are three possible cases. One is if the two curves intersect, as in Figure 3, and then the economy converges to a finite steady state, as in Figure 1, and the stationary level of technology and industrialization f^* is determined by the intersection. In the second case the R curve is everywhere above $1 + \rho$, namely $A \geq B(1+\rho)$ and there is unbounded growth, as in Figure 2. In the third case the R curve is everywhere below $1 + \rho$, namely $A \leq (1+\rho)m(0)$, then $f^* = 0$ and the economy is stagnant with no technical progress.

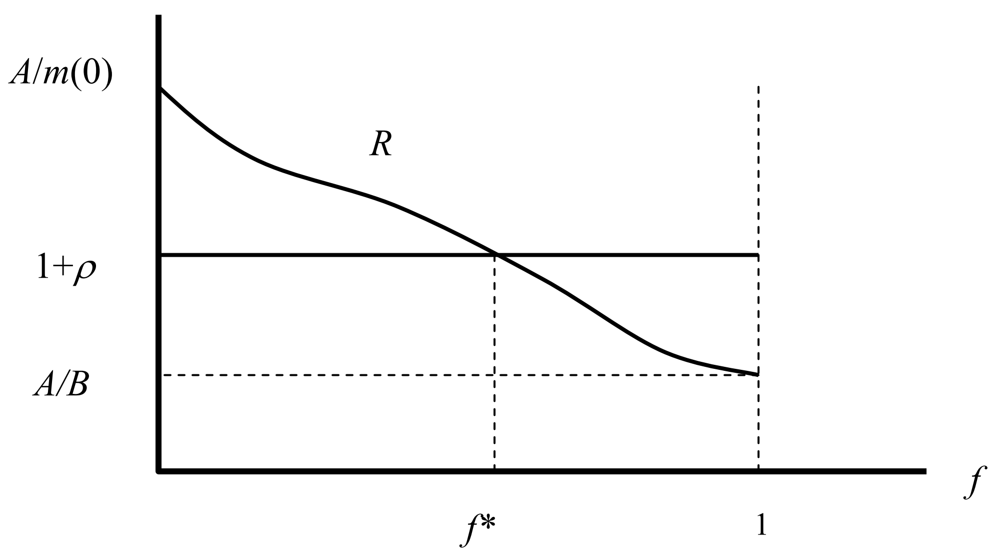


Figure 3: Determination of Long-Run Technology

Consider an economy that is in an initial pre-industrial steady state equilibrium in which output does not grow and no technology is invented. In terms of this model such a situation occurs when the R curve is everywhere below $1 + \rho$, so that $f^* = 0$. We next discuss a number of possible changes that could have triggered the industrial revolution.

5.1. A Decline in the Cost of Machinery

A first possible change is a reduction of the cost of machinery m, which reduces both $m(0)$ and B. As a result the curve R in Figure 3 shifts up, and if $(1+\rho)m(0) < A$, f^* increases and becomes positive. As a result the economy starts growing and begins to experience technical change, industrialization and economic growth. Its long run equilibrium depends on how large is the reduction in the cost of machinery. If $B/(1+\rho) > A$, the economy converges to a finite steady state, but if the reduction in the cost of machinery is more significant so that $B(1+\rho) \le A$, the economy experiences long-run growth. However in both cases growth begins and can continue for a long time.

What historical event could have lowered the cost of machinery so drastically? The most suitable candidate is the invention of the steam engine by the end of the eighteenth century. This was not only an invention of a single machine, but what we call a "general purpose technology." It signaled to all that there appeared a new way to produce goods, which is by machines instead of labor, by thermal energy instead of human energy. It also signaled that this new way was not too costly, but realistic and available. By reducing the cost of machinery, this invention pushed the economy into a path of industrialization and growth.

5.2. A Rise in Overall Productivity

A second exogenous change that can push the economy from a pre-industrial equilibrium into a growth path is a rise in overall productivity A. This productivity reflects climate, access to trade, infrastructure, etc. Such a rise in productivity A shifts the R curve in

Figure 3 upward and can also set the process of economic growth in motion, just as the reduction of the cost of machinery. If the rise in A is large enough, it can even lead to unbounded long-run growth.

What can be the historical equivalent of an increase in productivity prior to the industrial revolution? One possible event could have been the discovery of America, which contributed to sea faring, to agriculture, and also added new territories to Western Europe. As described by Maddison (2005): "World production potential was increased by an ecological transfer of plants and livestock across the Atlantic. The relative impact of this "Columbian Exchange" was greatest in the Americas, which acquired cattle, pigs, chickens, sheep, goats, wheat, rice, sugar cane, coffee, vegetables and fruits to enrich the diet, as well as horses and mules for transport and traction. There was a reciprocal transfer of New World's crops to Europe, Asia and Africa – maize, potatoes, sweet potatoes, manioc, tomatoes, peanuts, beans, pineapples, cocoa and tobacco – which enhanced the rest of the world's capacity to sustain population growth." As documented by Maddison (2005), these developments raised incomes: between 1500 and 1820 income per capita in Western Europe increased by more than 60%. Hence, the discovery of America could have been another potential trigger to the industrial revolution.

5.3. A Reduction of the Cost of Financial Intermediation

Our model assumes that all markets are perfectly competitive and especially that capital markets are perfect. But capital markets were only beginning to develop in the 17th and 18th centuries. Hence, it is reasonable to assume that the development of modern banking and large scale financial markets reduced the cost of financial intermediation, which could have been quite high prior to this development. This process reduced the cost of capital for investors significantly. A simple way to incorporate the cost of financial intermediation in the model is to add it to the discount rate ρ, so that the required interest rate become $1 + \rho + e$, where e is the cost of financial intermediation per unit of loan. This replaces the horizontal curve in Figure 3. In such an extension of the model a reduction of intermediation costs shifts the horizontal curve downward and thus can push the economy to growth and industrialization. Hence, this could be one of the possible triggers to the industrial revolution.

5.4. Decline in Monopoly Power

Another variable that can affect the growth path of the economy is the degree of competition in the economy, namely the extent of monopoly power. If producers have monopoly power, it increases their profits and thus inevitably reduces wages. Hence, monopoly power lowers wages and as a result reduces the incentive to industrialize. This suggests that a fourth possible trigger to the industrial revolution could have been the collapse of feudalism, which reduced monopoly power. This collapse started in England with the Cromwell Revolution, was accelerated by the spread of Protestantism, and was further intensified with the French Revolution and the Napoleonic wars. During the 19th century all over Europe the old system of control by few over land and production continued to crumble down. Our model indicates that such a historical development could also have been one of the possible triggers to the industrial revolution.

5.5. Some Recent Historical Findings

It is far beyond the scope of this paper to examine historically the role of each of these four developments as triggers to the industrial revolution. It points at the theoretical possibility that each of them separately, and all four together, could have acted as triggers to the industrial revolution and to the onset of modern economic growth. It is interesting though to compare the results of this model to recent research in economic history, mainly to Allen (2009). This book claims that the industrial revolution began in Britain in the turn between the 18th to the 19th centuries as a result of the prices of the factors of production. It shows that wages in Britain increased significantly during the 18th century, while the price of energy, mainly coal, declined during that period. Allen (2009) shows that these developments created strong incentives to develop technologies that replace human labor by an alternative factor of production, namely by machinery. This book clearly supplies important historical support to the main idea of the effect of labor cost on technology. But we can use this historical information to further distinguish between the four possible triggers discussed in this paper. Of the four, two do not predict any rise in wages prior to the onset of the industrial revolution, and these two are the decline in the cost of machinery and the improvement in financial intermediation. Only the two other possible triggers to the industrial revolution can predict a rise in wages prior to its beginning and these are the rise in overall productivity, due to the discovery of America, and the collapse of Feudalism. Note that these two events affected England earlier and more intensely than other countries. That can better explain why wages in England were higher than in other countries at the time, as shown by Allen (2009).

6. Fitting the Model with the Data

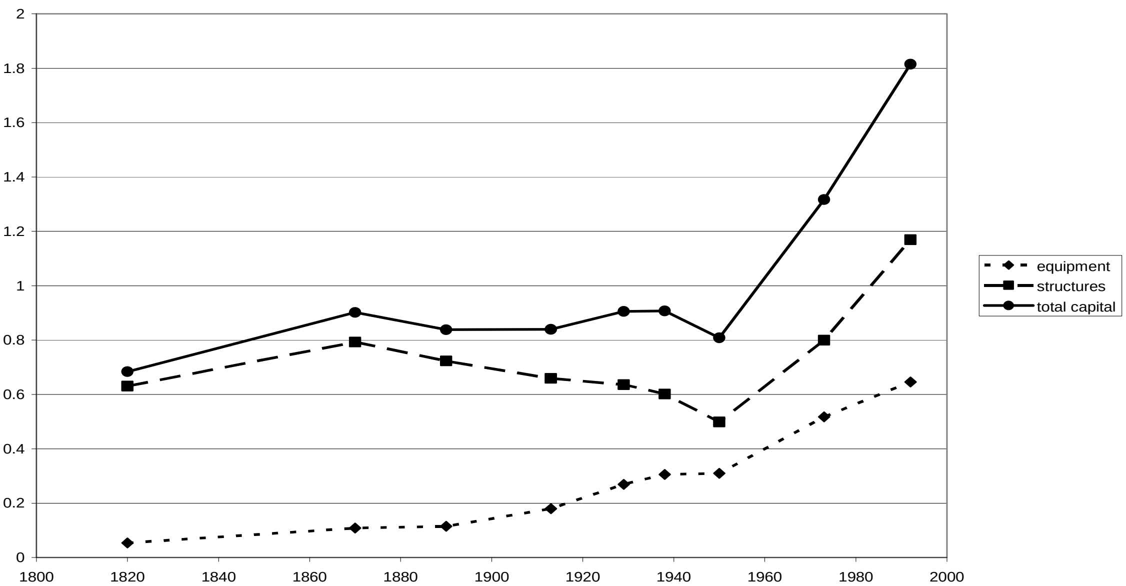
As the economy is growing and new machines are invented to perform a growing number of intermediate goods, the ratio of capital to output, which is equal to f_t/R_t, according to (17), increases over time. As already observed by Kaldor (1961), this ratio tends to be stable over time. Furthermore, this ratio is closely related to the share of capital in gross output, which is equal to f_t in this model, and which has also been found to be stable over time. The rise of the capital-output ratio is not specific to this model alone, but to all models where labor costs induce technical change. Hence, the need to reconcile the theory with the stylized facts is important.[8]

This section raises doubts about these stylized facts by presenting two empirical observations. First, that over a long period of time, from the industrial revolution, the ratio of capital to output in the leading countries was not stable and has increased significantly. Second, if we split capital to structures and equipment and machines, the empirical results are even further away from the standard "stylized facts." In these countries the ratio of equipment and machinery capital to output increased significantly and steadily with economic growth, while the ratio of structure capital to output fluctuated quite a lot. Hence, the overall dynamics of the ratio of capital to output were less pronounced than the dynamics of equipment and machinery to output.

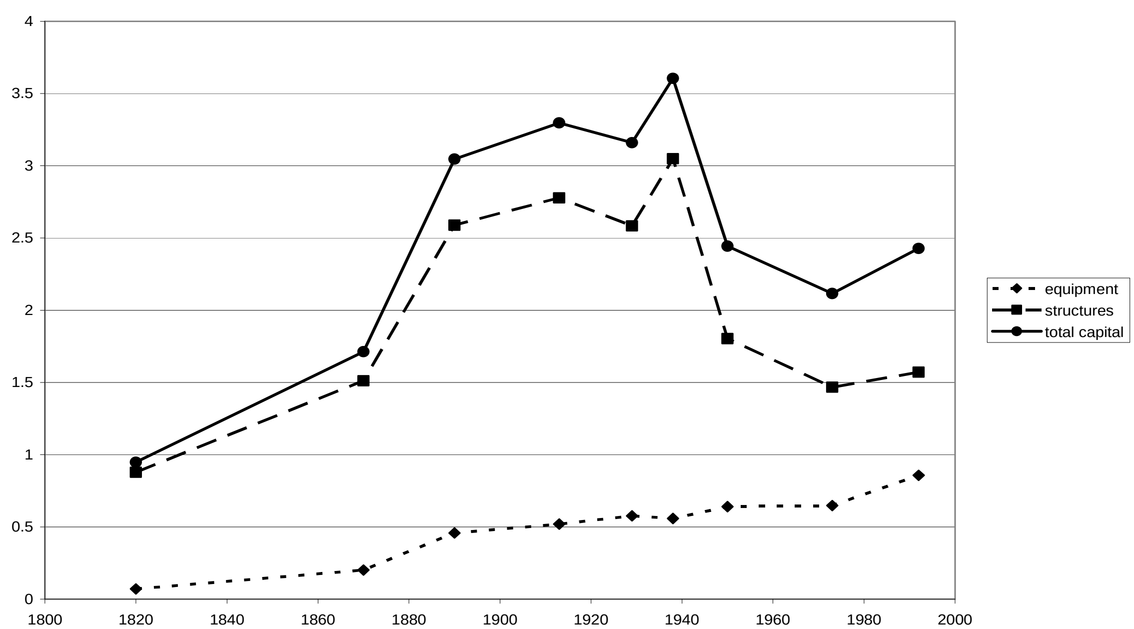
We turn first to data presented in Maddison (1995, Appendix K-1) on the ratio of capital to output. The three countries with the longest series of data are UK and US from 1820 and Japan from 1890. The following graphs in Figure 4 show the ratios of capital to output in selected years in these countries, both for total capital and for its breakup to equipment and structures. As Figure 4 clearly shows, over the last two centuries the capital output ratio increased significantly in all three countries and especially in Japan. The capital output ratio in the USA fluctuated significantly in the 20th century, but only due to fluctuations in structures. The ratio of equipment and machinery capital to output

[8] Zuleta (2006) shows that if there is an additional good in the economy, which is produced by labor only, like services, the model can be made to fit this stylized fact better. Peretto and Seater (2006) claim that if human capital is added to physical capital their ratio to output grows over time.

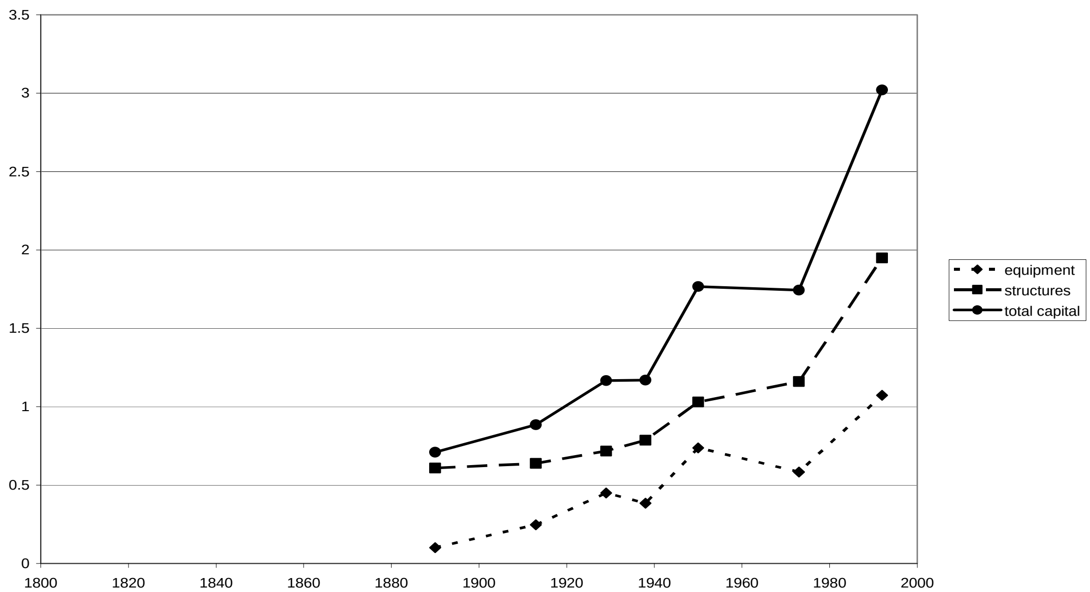
increased significantly during this period in all three countries. From 1820 till 1992 it rose from 0.05 to 0.65 in the UK, and from 0.07 to 0.86 in the US. In Japan it rose from 0.10 in 1890 to 1.07 in 1992.



A. UK



B. US



C. Japan

Figure 4: Capital-Output Ratios in UK, US and Japan

Maddison (1995) also presents data on capital-output ratios for three more countries: France, Germany and the Netherlands, but for a shorter period of time, from 1950 to 1992. The pattern of capital output ratios for these countries is the same as in Figure 4. In all three countries the ratio of capital to output increased in those years, in France by 39%, in Germany by 29% and in the Netherlands by 12%. But in those three countries machinery and equipment capital increased by much more relative to output. In France this ratio increased from 0.21 to 0.74, in Germany it increased from 0.39 to 0.70, and in the Netherlands from 0.27 to 0.78.

As Figure 4 shows, the rise in equipment capital to output was very significant in all three countries, UK, US and Japan, but in the US it slowed down significantly in the

20th century. We therefore try to focus more on the US in this period, and especially in the second half of the 20th century, as the first half was disrupted by two world wars and the great depression. We focus here on the private sector and examine data on the ratio of equipment capital to value added separately in industry and in services. The data are from 1947 to 2005 from the BEA (2008) and it is interesting to present them as the figures are quite different from those in Maddison. As shown in Figure 5 there is a strong rise in the ratio of equipment capital to value added in industry, while there is a much smaller rise in services. Since the share of services in output grew significantly during this period, the aggregate ratio of equipment capital to output did not rise by much.

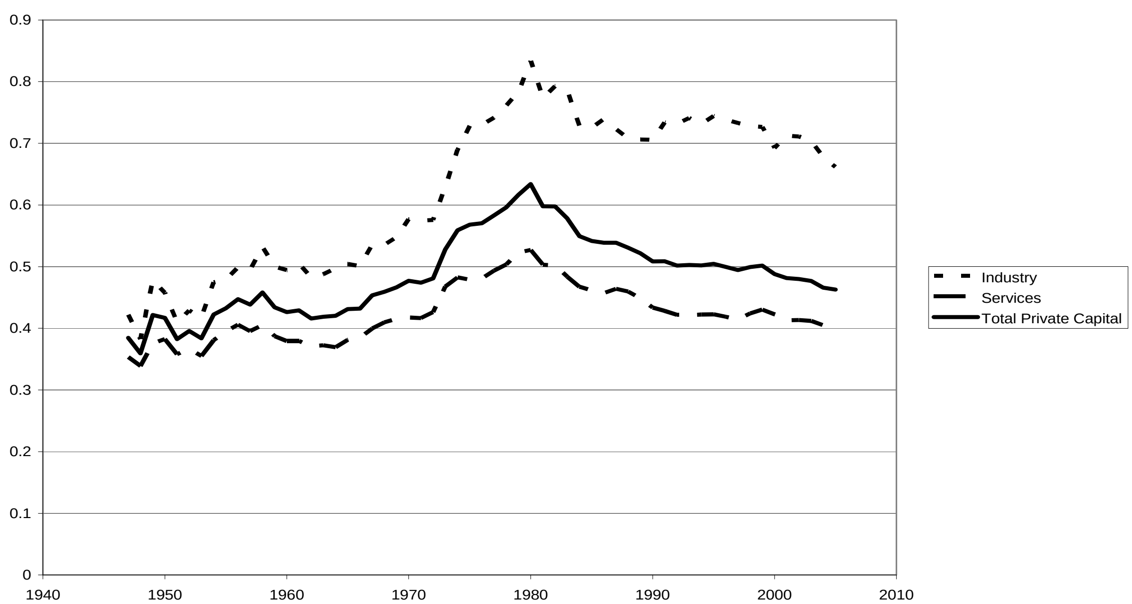


Figure 5: Ratios of Equipment Capital to Value Added in the US, 1947-2005

Sources: BEA (2008)

Figure 5 also shows that there was a significant change in the trend of equipment capital to output in the late 1970, which could be well explained by the oil shocks in that decade. Note that substituting labor by machines also changes the type of energy used, from human energy to fossil energy, coal, oil, etc. When energy becomes more expensive, the machines that replace labor become less desirable as a result. This can explain both the decline after 1980 in the ratio of equipment capital to output and also the strong shift from industry to services, which are both shown in Figure 5.

7. Comparison with Other Endogenous Growth Models

In this paper economic growth is driven by both capital accumulation and by continuous invention of machines. Hence it shares common elements with the two main theories of endogenous growth, namely the AK models and the R&D based growth models. But this paper also differs significantly from these two theories. This section discusses the similarities and the differences.

7.1 Comparison with AK Models

One possible interpretation of this model is that it supplies micro-foundations to the AK models, which were initiated by Jones and Manuelli (1990) and Rebelo (1991). These models assume that the economy grows along a constant production function, without technical change, and this is possible if the marginal productivity of capital is not diminishing to zero, namely if the Inada condition does not hold. Note that this requirement fits this model as well. The aggregate production function, which is discussed in Proposition 2, has a marginal productivity of capital which is indeed bounded from below by A/B.

Hence, this model has significant similarity to the AK models, but it differs from them in offering a micro-foundation story of innovations, which replace workers by machines in an ever increasing set of tasks of production. But this is more than supplying micro-foundation to an existing model, since in this model the dynamic implications can differ significantly than in standard AK model. This holds especially with respect to the Solow residual. In AK models it must be equal to zero, since the economy is growing within a stable production function, but this contrasts the fact that positive Solow residuals are well established empirically. But if in this model the set of technologies that constitute the aggregate production function are not always known, and are invented over time, due to any type of friction, then the Solow residual becomes positive as well. The intuitive reason is that in that case the long-run marginal productivity of capital deviates in one way or another from the short-run marginal productivity of capital, which is lower. This makes the theoretical Solow residual positive, as in reality and unlike the theoretical one in AK models.

There are many frictions that can lead to gradual invention of machines. The more reasonable ones are technological. This paper adopts a simple type of friction, namely adjustment costs to investment. Even that can lead to a positive Solow residual. Note that if we subtract adjustment costs from output, the share of capital becomes smaller and that increases the theoretical Solow residual. A calculation of this residual in this model leads to the following size:

$$\frac{\gamma^2 (q-1)^3}{2} \frac{f_t}{R_t}. \tag{27}$$

This is clearly positive along the growth path, unless the steady state is finite.

7.2. Costly Innovation: Comparison with R&D Based Growth Models

The similarity between this model and the R&D based endogenous growth models is obvious, since in both theories growth is driven by new innovations. But this model differs from these R&D models in assuming that the innovations are embodied in machines and thus the cost of the machines must be sufficiently low to create demand for the innovation. Existing R&D models focus mainly on the cost of the innovation itself, which is a fixed cost, and that is why they create such a strong scale effect. In this model this cost is ignored by assuming that it is equal to zero. We next show that even if this cost is positive, the scale effect is much weakened and might even disappear at a large scale. The intuitive reason is that if inventions are embodied in machines, the invention has not only fixed costs but variable costs as well and the larger the scale the smaller the effect of the fixed costs. This is an important, since the data does not exhibit significant scale effects, as shown by Jones (1995) and others.

In order to show it, consider the model presented in Section 2 with only one change, namely that inventing a new machine is costly. Let the cost of innovation depend, as in the R&D growth models, on the alternative cost of resources, namely on output per capita.[9] Formally, the cost of inventing a machine for task j in period $t-1$, which is when the machines used in t are invented, is:

$$\frac{bY_{t-1}}{N}. \tag{28}$$

For simplicity assume that a patent lasts only one period and in next periods it becomes public knowledge. Hence, machine j, costs $m(j)$ in future periods, but $m(j)+z_{t-1}$ in its first period, t–1, where z is the patent fee.

Due to competition between innovators the patent fee for an invented machine is equal to the cost of innovation divided by the amount of machines sold:

[9] This is of course a simplifying assumption. Alternative assumptions on cost, like wages, yield similar results.

$$z_{t-1} = \frac{bY_{t-1}}{NX_t(j)} = \frac{bY_{t-1}}{N\dfrac{Y_t}{R_t[m(j)+z_{t-1}]}} = \frac{bR_t[m(j)+z_{t-1}]}{N(1+g_t)}.$$

From this condition we can derive the value of the patent fee:

(29) $$z_{t-1} = \frac{bR_t m(j)}{N(1+g_t) - bR_t}.$$

The first order condition with respect to the frontier of technology is equal in this case to

(30) $$\frac{w_t}{R_t} = m(f_t) + z_{t-1} = m(f_t)\left(\frac{N(1+g_t)}{N(1+g_t) - bR_t}\right).$$

This condition implies that with costly innovations there exists a scale effect, namely a larger scale *N* can increase innovations and growth. But this scale effect is diminishing, and as *N* becomes really large, the scale effect becomes negligible and equation (30) converges to equation (11). Thus, the scale effect in the original R&D based growth models, which does not fit the data well, as shown by Jones (1995a), is much reduced in this model. Therefore, scale can contribute to economic growth, but only to a limited and diminishing extent.

8. Summary

This paper presents a model of industrialization, and describes it as a process of inventing new machines that replace labor in a growing set of tasks. In this process the wage rate plays a crucial role. Wages serve as an incentive for adopting new technologies. But wages are also positively affected by these technologies, since workers who operate with more machines become more productive. This feedback between wages and technology is the main mechanism that drives the results of this paper. It explains how the growth process can continue for a long time, it explains how growth is so sensitive to the cost of machinery, to productivity, and it also explains why monopoly deters growth.

Finally, it is time to briefly discuss the type of innovations in this paper, namely machines that replace human labor. Although this is only one specific type of innovation, it can be shown to be quite common and general. Even an innovation that replaces a machine by a better machine enables the workers who operate it to use less labor in production. Furthermore, even innovations of new consumption goods tend to replace labor in one way or another. A dishwasher, TV dinner, radio, cinema, all replace labor, either at home, or in the workplace. We do not have to go back in history to the Ludites, to realize that new machines that replace human labor have played a central role in economic growth since the industrial revolution. This paper shows that embodying this insight into growth theory can help us to better understand the process of economic growth.

Appendix

Proof of Proposition 2:

Maximum output $F(K,L)$ is defined by:

(A1) $$F(K,L)=\max_{I,X(j)}\left\{\exp\left[\log A+\int_0^1 \log X(j)dj\right]: K=\int_I m(j)X(j)dj, L=\int_{I^c} X(j)dj\right\}.$$

First, this maximization implies that the set of industrial tasks includes the least costly machines, so $I=[0,i]$. Hence, output maximization can be rewritten as:

(A2) $$F(K,L)=\max_{i,X(j)}\left\{\exp\left[\log A+\int_0^1 \log X(j)dj\right]: K=\int_0^i m(j)X(j)dj, L=\int_i^1 X(j)dj\right\}.$$

Let z_1 and z_2 be the two Lagrange multipliers for the two constraints in (A2) respectively. Then the first order conditions are the following:

(A3) $$\frac{Y}{X(j)}=z_1 m(j),$$

if $0\le j\le i$,

(A4) $$\frac{Y}{X(j)}=z_2,$$

if $i<j\le 1$, and

(A5) $$m(i)=\frac{z_2}{z_1}.$$

The budget constraints are also satisfied:

(A6) $$L=\int_i^1 X(j)dj=(1-i)\frac{Y}{z_2},$$

and:

(A7) $$K = \int_0^i m(j)X(j)dj = i\frac{Y}{z_1}.$$

Note that equations (A3)-(A7) are identical to equations (9)-(13) and the production (1) as well. Hence the two sets of equations lead the same solutions if $K = K_t$ and $L = N$. We therefore get $Y_t = F(K_t, N)$, and also: $w_t = z_2$ and $R_t = z_1$. Note that according to the Kuhn-Tucker theorem the marginal derivatives of *F* are equal to the Lagrange multipliers, and hence:

(A8) $$MPK = \frac{\partial F(K_t, N)}{\partial K} = z_1 = R_t,$$

and:

(A9) $$MPL = \frac{\partial F(K_t, N)}{\partial L} = z_2 = w_t.$$

As shown above the equilibrium output is equal to the optimum quantity of output given the inputs and that means that production is optimal. Next we show that capital accumulation is optimal as well. Consider a central planner who maximizes the representative consumer utility, which is the sum of discounted consumption, according to (5):

(A10) $$\max_{\{K_t\}} \left\{ \sum_{t=0}^{\infty} \frac{F(K_t, N) - K_{t+1} - \frac{1}{2\gamma}\frac{(K_{t+1} - K_t)^2}{K_t}}{(1+\rho)^t} \right\}.$$

The first order condition of maximization of (A10) with respect to K_{t+1}, using (A8), is:

(A11) $$-1 - \frac{1}{\gamma}\frac{K_{t+1} - K_t}{K_t} + \frac{1}{1+\rho}\left[R_{t+1} + \frac{1}{2\gamma}\frac{(K_{t+2} - K_{t+1})^2}{K_{t+1}^2} + \frac{1}{\gamma}\frac{K_{t+2} - K_{t+1}}{K_{t+1}} \right] = 0.$$

Let us define a variable p_t in the following way:

(A12) $$\frac{K_{t+1} - K_t}{K_t} = \gamma(p_t - 1).$$

Substituting (A12) in (A11) we get:

(A13) $$p_t(1+\rho) = R_{t+1} + \frac{\gamma}{2} p_{t+1}^2 + (1-\gamma) p_{t+1} - 1 + \frac{\gamma}{2}.$$

Note that equations (A12) and (A13) are identical to the dynamic equations of equilibrium, (19) and (21), if $p_t = q_t$. Hence, the optimal capital accumulation is identical to the equilibrium. It follows that the equilibrium path of growth and technology is optimal. QED.

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