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Economic growth and sector dynamics

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ABSTRACT

This paper analyzes the endogenous determination of sectors in a growing economy. It assumes that there are traditional sectors and modern sectors and economic growth is driven by rising productivity of the modern sectors. It also assumes that individuals are heterogeneous, which leads to increasing marginal opportunity costs in creating new modern sectors. We show that under these main assumptions, economic growth first increases diversification to sectors and then reduces it. We also show that for the equilibrium to be stable and well-behaved, it is required that the modern and traditional sectors should be substitutes and not complements.

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1. Introduction

Recent decades have seen a dramatic increase in research on economic growth and development¹. Naturally, this literature is mainly macroeconomic and it focuses on how aggregate output rises over time and why it differs across countries. But economists have always known that the process of economic growth is not only aggregate, but entails deep structural changes over time. The obvious and most famous one is the move from agriculture to industry and later from industry to services². However, the structural changes that come with development are much wider than this change alone. Economic growth changes continuously the set of goods produced and the set of goods consumed. Therefore, growth leads to continuous changes in the diversification to sectors. Unfortunately, this process has not received sufficient attention in the wide research on economic growth³.

This paper contributes to this area of research by introducing a model that connects the dynamics of economic growth to the dynamics of the sector structure in the economy. The model has two types of sectors, traditional and modern, and

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¹ This broad area of research is surveyed in two volumes of the Handbook of Economic Growth, Aghion and Durlauf (2005, Acemoglu and Zilibotti (2014).

² See early work by Kuznets (1966).

³ Only one chapter in the two volumes of the Handbook of Economic Growth, Herrendorf et al. (2014), discusses sectors. It focuses on the three main sectors agriculture, industry and services.

economic growth is driven by increasing productivity in the modern sectors. As a result economic growth leads to transition of production from the traditional to the modern sectors, where the number of traditional sectors declines and the number of modern sectors increases. The model reaches two main results. The first is that sector diversification in the economy follows an inverse *U* shaped curve along economic growth, first rising and then declining. This result is interesting and important because it fits well the empirical findings of Imbs and Wacziarg (2003). The second main result of the model is that the elasticity of substitution between the modern and traditional goods should be greater than one for the equilibrium to be well behaved. We also show that due to similar reasons the elasticity of substitution between the modern sectors themselves and between the traditional sectors themselves should be even higher.

The model in this paper builds on a number of assumptions, some are crucial for the main results and some are only made for simplification. First, we assume that there are two types of sectors, traditional and modern. Second, it is assumed that each type employs a different type of labor, raw labor in traditional sectors and efficiency labor in modern sectors. A third main assumption is that efficiency labor, which is used in modern sectors, differs across people. The fourth assumption, which is crucial to our results, is that productivity of modern sectors rises exogenously over time and this is the engine of growth in the model. We treat technical change as exogenous, as we view countries as small open economies that use mainly technologies that are invented elsewhere⁴. The other assumptions are less crucial for the main results of the paper. We assume that there is some size requirement to production, in order to avoid the case of infinite number of sectors. Two types of such requirements are considered, one at the firm level, in the benchmark model, and one at the sector level in an extension in Section 6.1. While these two alternative assumptions imply different market structures, monopolistic competition and perfect competition, respectively, they do not affect the main results of the model. The model also treats the products of the various sectors as intermediate inputs in the production of the aggregate good, and not as consumption goods. This assumption as well does not have significant effects on the main results of the model.

Sector dynamics in this paper can therefore be described as follows. As productivity in modern sectors increases, income in these sectors rises, so more workers choose to supply efficiency labor instead of raw labor and move to modern sectors. New modern firms are created and instead of locating in existing sectors, where higher output lowers the price, firms prefer to open new modern sectors, where prices are higher⁵. Hence, the number of modern sectors increases and the number of traditional sectors declines. As more people supply efficiency labor, its average efficiency declines, since efficiency labor is supplied by people with the highest efficiency. Hence, each new modern sector requires resources from more traditional sectors. As a result the marginal opportunity cost, in terms of traditional sectors, of building a new modern sector is increasing. This affects the total number of sectors. As a new modern sector is set, the total number of sectors increases by one minus the marginal opportunity cost of this sector. As long as this marginal cost is lower than one, the total number of sectors increases with economic growth. Once the marginal cost exceeds one, the total number of sectors begins to decrease. Hence, diversification to sectors increases up to some level of development and from there on it declines. This result, of an inverted *U* path of sector diversification along economic growth, is an important result of the paper.

As mentioned above, this result fits well the empirical study of Imbs and Wacziarg (2003), who find that diversification to sectors first increases with economic growth up to some level of per capita income, but beyond it sector diversification tends to decline. This result holds both over time and across countries. Our explanation to this inverse *U* path of diversification relies on a very basic assumption in economics, namely increasing marginal costs. The second main result of the paper is substitutability between sectors. In the solution of equilibrium of the model, we realize that to have a stable and well-behaved equilibrium we need to add some restrictions on the main parameters of production. These restrictions imply that the elasticity of substitution between modern and traditional goods should be higher than one and also that the elasticity of substitution between the modern intermediate goods themselves and between the traditional intermediate goods themselves should be even higher than the substitution between the modern and the traditional goods. Note that modern and traditional productions require different types of labor, which can be also correlated with education. Interestingly, empirical studies, summarized in Caselli and Coleman (2006), have shown that the elasticity of substitution between high-educated and low-educated labor is around 1.4. Since the elasticity of substitution between the relevant goods should be close to it, we can view it as some supporting evidence.

There are two main literatures that are closely related to our paper. The first literature is on sector diversification, and the second literature is on diverging growth paths across sectors. The first literature, on diversification, can be divided to two separate strands, where each highlights a different relation between growth and diversification. Dornbusch et al. (1977) predict a negative monotonic relation between development and diversification, since reduction in transport costs reduces the set of non-traded goods and leads to concentration. Krugman (1991) also predicts a negative relation between growth and diversification, but the mechanism he suggests is geographic agglomeration. Another part of this literature reaches an opposite prediction, namely that income and diversification should be positively related. Matsuyama (2000) assumes non-homothetic preferences, so when income rises, agents spend less on each good and new sectors emerge. Acemoglu and Zilibotti (1997) view sectors as risky projects, which have a minimum size. Hence, higher income enables a greater diversification of risk, which implies more sectors. Regev and Zoabi (2014) also predict a positive relation between income

⁴ In Section 2 we elaborate more on this assumption and supply evidence to support it.

⁵ This process resembles the literature on diversification to states, as described in Alesina and Spolaore (1997) and Acemoglu and Wacziarg (1998).

and diversification, but their mechanism builds on talent utilization. This paper differs from the rest of this literature as it reaches a result of non-monotonic relation between growth and diversification.

Another literature studies unbalanced growth of sectors. It mainly tries to explain the dynamics of the shares of the three main sectors, agriculture, industry and services. This paper is related to this literature, since it also has two types of sectors, traditional and modern, which grow at different rates. This literature can also be divided to two opposing lines of research. In the first the main sector with faster technical change attracts more labor and becomes dominant. Echevarria (1997), Laitner (2000), Kongsamut et al. (2001), Gollin et al. (2002) and Foellmi and Zweimuller (2008) are examples of such models. In the second line of research the low growing main sector attract most labor and dominates the economy. Examples of such results are Baumol (1967), Caselli and Coleman (2001), Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008). Due to our result of high substitutability of the two main goods, this paper is closer to the first line of research. This result of high substitutability is explained below in the paper as a result of the types of these sectors and the types of inputs used in them.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 derives the main equilibrium conditions. Section 4 derives the general equilibrium and examines the implications of having a well-behaved and stable equilibrium for the substitutability between the two types of goods. Section 5 discusses the relation between growth and diversification. Section 6 analyzes two extensions of the model. Section 7 concludes. The Appendix contains proofs.

2. The model

Consider an economy that produces one final good Y , which is used for consumption only. The final good is produced by two aggregate goods: a traditional good, T , and a modern good, M . Each of the two aggregate goods is produced by a discrete number of intermediate goods, each produced in a separate sector. The numbers of the intermediate goods, J_T and J_M , which are also the numbers of sectors of each type, are determined endogenously. There are two main factors of production: raw labor and efficiency labor. While traditional sectors use raw labor, modern sectors use efficiency labor. The markets for the two types of labor and for the aggregate goods, which are the final, the traditional and the modern goods, are perfectly competitive. As shown below competition within sectors is not perfect. The economy is open and small. While the final good is tradable, labor as well as intermediate goods are non-tradable and their markets are domestic.

2.1. Individuals

Individuals live one period each in non-overlapping-generations. Assume that there is no population growth and the population size in each generation is L . Each individual is endowed with one unit of raw labor and is also endowed with a random amount of efficiency labor. This amount of efficiency labor, denoted e , is uniformly distributed across people over $[0, E]$. Each person chooses whether to supply raw labor or efficiency labor but she cannot supply both. People maximize utility: $u = c$, where c is consumption of the final good. Hence, each person chooses the type of labor that maximizes her income. In each sector labor can be hired for production or for expertise, where experts acquire 'know-how' of their respective sector. We assume that this acquisition is costless, so that workers and experts are perfectly substitutable and earn the same income⁶. This holds both for raw labor in traditional sectors and for efficiency labor in modern sectors. For the sake of simplicity we assume that expertise is independent of the random level of efficiency labor.

2.2. Production of the final good, the traditional good and the modern good

The final good is produced from the traditional and the modern aggregate goods according to the following CES production function:

$$Y = (Y_T^\rho + Y_M^\rho)^{\frac{1}{\rho}}. \quad (1)$$

Y is the quantity produced of the final good, while Y_T and Y_M are the quantities of the traditional and the modern goods, respectively. The traditional good itself is produced by intermediate goods from the traditional sectors according to:

$$Y_T = \left(\sum_{j=1}^{J_T} y_{Tj}^\rho \right)^{\frac{1}{\rho}}. \quad (2)$$

In this CES production function y_{Tj} is the quantity of intermediate good j produced in traditional sector j . Similarly, the aggregate modern good is produced by the following CES production function, where y_{Mj} is the quantity of intermediate

⁶ In Section 6.1 we examine the case of costly acquisition of 'know-how'.

1 good j produced in modern sector j :

$$3 \quad Y_M = \left(\sum_{j=1}^{J_M} y_{M,j}^\sigma \right)^{\frac{1}{\sigma}}. \quad (3)$$

5 The elasticity of substitution between the traditional good and the modern good is $1/(1-\rho)$, while the elasticity of substitution between traditional intermediate goods is $1/(1-\sigma)$ and this is also the elasticity of substitution between modern intermediate goods. As is usually assumed on CES production functions, ρ and σ are between $-\infty$ and 1. We do not impose more restrictions on these parameters here, but the analysis below leads to additional restrictions on them, which are required to ensure a stable and well-behaved equilibrium.

11 2.3. Production in sectors

13 Each of the traditional and modern intermediate goods is produced in a separate sector by production workers and by experts. Each firm i in a traditional sector j requires expertise at a quantity x of raw labor at any level of production. Its output depends on the amount of raw labor in production $l_{j,i}$ as follows:

$$17 \quad y_{T,j,i} = l_{j,i}^\alpha. \quad (4)$$

19 The parameter α is between 0 and 1. Each firm i in a modern sector j also requires expertise at a quantity x of efficiency units of labor for any level of production. Its output depends on the amount of efficiency labor in production $h_{j,i}$ in the following way:

$$23 \quad y_{M,j,i} = Ah_{j,i}^\alpha. \quad (5)$$

25 The parameter A is the productivity of modern sectors. A can be also interpreted as the state of technology in the country, which is common to all modern sectors and is further discussed in the next sub-section⁷.

27 2.4. Technical change

29 The model's main engine of growth is technical change, which applies to modern sectors only. We also assume that technology A is exogenous for the country. These two assumptions require some explanation. The assumption that growth is mainly a result of technical change is fairly realistic. Usually economists consider both technical change and acquisition of human capital to be the two main engines of growth. But historical evidence shows that accumulation of human capital accounts for only a small part of economic growth in the last two centuries⁸. The assumption that technical change is exogenous can be justified by two arguments. First, since most R&D is done by a small group of countries, especially the US, we can assume that there is a global technological frontier, which increases over time, and countries just follow it, treating it as exogenous. But some countries do not follow this global frontier fully, as is well documented in the literature⁹. In this paper we assume that this partial adoption of technology is exogenous as well. This is a simplifying assumption, but it is also based on some supporting evidence. A recent empirical study by Battisti et al. (2014) finds that the degree of following the frontier across countries depends on variables like climate, ethnic diversity or openness to trade, but not on sector structure. Thus, in a model that focuses on sector dynamics, we can assume that technical change is exogenous.

31 3. Equilibrium conditions

33 3.1. Profit maximization in production of the final, traditional and modern goods

35 Let the price of the final good be normalized to one and the prices of the traditional and the modern goods be denoted by P_T and P_M , respectively. The prices of the intermediate goods, traditional and modern, are denoted by p_{Tj} and p_{Mj} , respectively. The wage of raw labor is w_l , and this is also the wage of traditional experts, since we assume that acquisition of 'know-how' is costless. The wage of one unit of efficiency labor is w_h , and this is also the wage of an efficiency unit hired for expertise in a modern sector. Profit maximization by producers of the final good leads to the following first-order conditions:

$$37 \quad P_T = Y^{1-\rho} Y_T^{\rho-1},$$

39 ⁷ We assume for simplicity that traditional and modern sectors are quite symmetric, except for productivity. These assumptions are made for simplification only and do not affect the results at all.

41 ⁸ According to Maddison (2005) GDP per capita in the developed countries increased 20 times between 1820 and 2001. Average schooling increased by 10 years during that time, so by standard development accounting calculations, human capital increased only 3 times. See also Jones (2015).

43 ⁹ For empirical evidence on differences in technology across countries see Acemoglu and Zilibotti (2001), Caselli (2005), Caselli and Coleman (2006) and Comin and Hobijn (2011). For explanations for such differences see Parente and Prescott (1994), Zeira (1998), Basu and Weil (1998), and Brunt and Garcia-Penalosa (2011).

and:

$$P_M = Y^{1-\rho} Y_M^{\rho-1}.$$

We next turn to profit maximization by producers of the traditional and the modern goods. The profits of producers of the traditional good are:

$$P_T \left(\sum_{j=1}^{J_T} y_{Tj}^{\sigma} \right)^{\frac{1}{\sigma}} - \sum_{j=1}^{J_T} p_{Tj} y_{Tj}.$$

Hence, the first order condition with respect to input of intermediate traditional good j is:

$$p_{Tj} = P_T Y_T^{1-\sigma} y_{Tj}^{\sigma-1} = Y^{1-\rho} Y_T^{\rho-\sigma} y_{Tj}^{\sigma-1}. \quad (6)$$

This first order condition describes the inverse demand for the traditional intermediate good j .

Eq. (6) implies that the price is falling with the quantity of y_{Tj} purchased, since $\sigma - 1 < 0$. That creates an incentive for firms to prefer to create a new sector, rather than enter a sector that already exists, where there is at least one firm, since the price of the intermediate good will be higher in the new sector¹⁰. As a result, there is only one firm in each sector in equilibrium and it is therefore a monopolist. This result holds only if there is demand for a new intermediate good by the producer of the traditional good. To examine it calculate the derivative of the above profits with respect to the number of goods J_T and get

$$P_T \frac{1}{\sigma} Y_T^{1-\sigma} y_{Tj}^{\sigma} - p_{Tj} y_{Tj} = \left(\frac{1}{\sigma} - 1 \right) P_T Y_T^{1-\sigma} y_{Tj}^{\sigma}.$$

This implies that if σ is negative, producers of the traditional good T do not buy an additional intermediate good at the market price, since it reduces their profits. As a result, if $\sigma < 0$ the number of traditional sectors in equilibrium will be 1, so that a theory of sectors becomes redundant. The same problem arises with respect to modern sectors. To avoid such an outcome, we therefore add the following restriction:

Restriction 1. The parameter σ is positive: $\sigma > 0$.

Hence, every traditional sector has only one firm, which is a monopoly and this result holds also for modern sectors, as **Restriction 1** holds for them as well¹¹. In these sectors the first order condition is derived similarly and it is:

$$p_{Mj} = P_M Y_M^{1-\sigma} y_{Mj}^{\sigma-1} = Y^{1-\rho} Y_M^{\rho-\sigma} y_{Mj}^{\sigma-1}. \quad (7)$$

The result that each sector contains only one firm, which is a monopoly, depends not only on **Restriction 1**, but also on our assumption that 'know-how' acquisition is costless. If such acquisition is costly and these costs are mitigated by flows of 'know-how' between firms in a sector, production at the sector level will be characterized by increasing returns to scale. **Section 6.1** shows that in such a case, there can be many firms and as a result perfect competition in each sector.

3.2. Profit maximization in individual sectors

Profits of the only traditional firm in traditional sector j , which is a monopolist, are:

$$\pi_{Tj} = p_{Tj} y_{Tj} - w_l l_j - w_x x.$$

Applying the demand price of this intermediate good from Eq. (6) and the production function (4) we get that profits are equal to

$$\pi_{Tj} = Y^{1-\rho} Y_T^{\rho-\sigma} y_{Tj}^{\sigma} - w_l l_j - w_x x = Y^{1-\rho} Y_T^{\rho-\sigma} l_j^{\alpha\sigma} - w_l l_j - w_x x. \quad (8)$$

Maximization of monopolistic profits yields the following first order condition:

$$\alpha\sigma Y^{1-\rho} Y_T^{\rho-\sigma} l_j^{\alpha\sigma-1} = w_l. \quad (9)$$

Substituting this first order condition in (8), we get that the maximized profits in a traditional sector are:

$$\pi_{Tj} = w_l \left(\frac{1-\alpha\sigma}{\alpha\sigma} l_j - x \right). \quad (10)$$

¹⁰ We assume implicitly that the numbers of sectors, J_T and J_M , are high, so the effect of opening a new sector on Y_T is negligible. This is also why we treat the variables J_T and J_M as real numbers in the following analysis.

¹¹ This result relates our paper to [Murphy et al. \(1989\)](#), which also has sectors with monopolistic competition, but apart from that the papers are very different.

Similar profit maximization in modern sectors yields the following first order condition that describes the wage of efficiency labor:

$$\alpha\sigma Y^{1-\rho} Y_M^{\rho-\sigma} A^\sigma h_j^{\alpha\sigma-1} = w_h. \quad (11)$$

A similar calculation of profits shows that the maximized profits in a modern sector are:

$$\pi_{M,j} = w_h \left(\frac{1-\alpha\sigma}{\alpha\sigma} h_j - x \right). \quad (12)$$

3.3. Zero profit conditions

As long as profits of each sector are positive, there are incentives to enter and create new firms, namely new sectors. This entry continues until profits are equal to zero. From the profit Eqs. (10) and (12) it follows that the zero profit conditions determine the amounts of production labor in traditional and in modern sectors, which happen to be the same:

$$l_j = h_j = \frac{x\alpha\sigma}{1-\alpha\sigma}. \quad (13)$$

This equilibrium condition enables us to understand the role of the requirement of x experts in each firm. It sets a size for the firm and thus for the sector. This size is also the reason why we end up with a finite number of sectors, since otherwise the assumption that $\sigma > 0$ might lead to infinite number of sectors.

Eq. (13) also implies that all traditional sectors hire the same amount of labor l and thus each sector produces the same amount of output:

$$y_{T,j} = y_T = \left(\frac{x\alpha\sigma}{1-\alpha\sigma} \right)^\alpha.$$

Similarly all modern sectors hire the same amount of efficiency labor h and each sector produces the same amount of output:

$$y_{M,j} = y_M = A \left(\frac{x\alpha\sigma}{1-\alpha\sigma} \right)^\alpha.$$

3.4. Labor markets' equilibrium conditions

Individuals are identical with respect to their raw labor but differ in their efficiency labor endowments. Since their income in the modern sectors is proportional to this endowment, people with relatively low levels of efficiency labor go to work in the traditional sectors, either as producers or as experts, while people with high levels of efficiency labor go to work in modern sectors, as producers or as experts. We denote by s the threshold level of efficiency above which people work in modern sectors and below it in traditional ones. At s an individual is indifferent between working in the two types of sectors, hence the following condition holds:

$$w_l = w_h s. \quad (14)$$

We next turn to market clearing conditions of the two types of labor. We derive in Eq. (13) the number of workers in each traditional sector and the amount of efficiency labor in each modern sector. To that we should add the amount of expertise in each sector. Together we get the total amount of employment in each sector, which we denote by m :

$$m = x + \frac{x\alpha\sigma}{1-\alpha\sigma}.$$

Multiplying employment by the numbers of traditional and modern sectors yields the demands for raw labor and for efficiency labor, respectively. The supplies of the two types of labor are determined by the threshold efficiency level s between the two types of labor and by our assumption on the uniform distribution of efficiency labor¹². Hence, the labor market clearing condition for raw labor is:

$$J_T m = \int_0^s \frac{L}{E} de = \frac{L}{E} s. \quad (15)$$

¹² The results would be similar if the distribution of efficiency would be different.

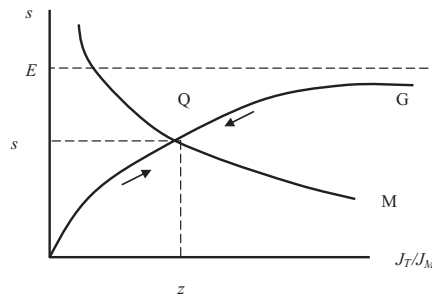


Fig. 1.

Similarly, the market clearing condition for efficiency labor is:

$$J_M m = \int_s^E \frac{L}{E} de = \frac{LE^2 - s^2}{E - 2}. \tag{16}$$

4. General equilibrium

In this section we put together the equilibrium conditions and show how they determine general equilibrium in the economy. This is done in a few steps. We first use the equilibrium conditions to describe the joint determination of two variables, the threshold between the two types of labor s , and the relative size of the two types of sectors, the ratio between the number of traditional sectors and the number of modern sectors, J_T/J_M . We show that in order to have out-of-equilibrium stability in each period and for the equilibrium to be well-behaved in very intuitive ways, we need to add some more restrictions on the parameters ρ and σ . We then show how the numbers of each type of sectors, J_T and J_M , are determined.

4.1. Determination of the threshold between raw and efficiency labor

We begin the derivation of general equilibrium by dividing the labor markets clearing conditions, Eqs. (15) and (16), by one another. We get the following condition that reflects equilibrium in the two labor markets:

$$\frac{J_T}{J_M} = 2 \frac{s}{E^2 - s^2}. \tag{17}$$

This positive relationship is described by the curve G in Fig. 1 below, which is upward sloping but bounded by E .

Another relationship between these two variables is derived from Eq. (14), according to which the threshold efficiency level s should be equal in equilibrium to the wage ratio between the two types of labor, due to mobility between them. This ratio between the wage of raw labor and the wage of efficiency labor can be calculated by using the first order conditions, (9) and (11), which yield after some manipulation:

$$s = \frac{w_l}{w_h} = A^{-\sigma} \left(\frac{Y_T}{Y_M} \right)^{\rho - \sigma} = \left(\frac{J_T}{J_M} \right)^{\frac{\rho - \sigma}{\sigma}} A^{-\rho}. \tag{18}$$

To simplify notation denote from here on $\chi = (\rho - \sigma)/\sigma$. Eq. (18) is drawn in Fig. 1 as the curve M , since it describes labor mobility between the two types of sectors. The specific curve M in Fig. 1 is drawn under the assumption that χ is negative. Equilibrium of the economy requires that both (17) and (18) hold, namely that the two curves, G and M , intersect. We denote this equilibrium point in Fig. 1 by Q and it determines the equilibrium threshold between raw and efficiency labor s and the ratio between the sizes of the two aggregate sectors, J_T/J_M , which we denote by z .

We first examine the existence of equilibrium. If χ is negative, as in Fig. 1, the equilibrium always exists and is unique, since the M curve goes from infinity at left to zero at right. If $\chi = 0$ there is also a unique equilibrium, while if $\chi > 0$ the equilibrium might not be unique, and this case is discussed in Section 4.2 and in Appendix 1.¹³ We next analyze out-of-equilibrium stability of equilibrium by examining location relative to curve M . If the economy is above M , namely if $s > w_l/w_h$, the marginal individual earns more from efficiency labor than from raw labor, so people move from raw to efficiency labor, and s is reduced. Similarly, below M , the threshold s is increasing. Hence, only an equilibrium point, in which M intersects G from above, is stable. Clearly if χ is negative, as assumed in Fig. 1, the unique equilibrium point is stable and

¹³ If $\chi = 0$, the M curve is horizontal and the two curves have a unique intersection if $A > E^{-1/\rho}$. If $A \leq E^{-1/\rho}$, equilibrium is reached at $s = E$.

1 this holds also for the case of $\chi = 0$. The following subsection explains why we restrict the model to non-positive χ , and it
 3 also adds a restriction on the parameter ρ .

5 4.2. Further restrictions on the parameters and substitutability

7 **Appendix 1** shows that if χ is positive the equilibrium is either unstable, or it leads to production only in traditional
 9 sectors for any level of productivity A , or it is not continuous with changes in A . All these three outcomes are problematic
 11 and a theoretical model of sector dynamics should avoid them. First, we would like to have a stable equilibrium. Second, it is
 13 not reasonable to have equilibrium with only traditional sectors, if the productivity of modern sectors is very high. Third,
 15 equilibrium is discontinuous with respect to A because the economy produces only in traditional sectors if A is low, but at
 some level of A it jumps and begins to produce at a positive number of modern sectors. Such discontinuity should also be
 avoided in a well-behaved theoretical model. As a result of these considerations we restrict the model from here on in the
 following way:

17 **Restriction 2.** The parameter χ is non-positive, which means: $\rho \leq \sigma$.

Note that changes in A have no effect on the G curve, but they shift the M curve in a way that depends crucially on the
 sign of the parameter ρ . Assume first that ρ is negative. In this case, if productivity of modern sectors A is reduced to zero,
 the M curve shifts down and the equilibrium s falls to zero. This means that more and more workers move to the modern
 sectors as they become less and less productive. In the limit, when their productivity is zero, the economy is producing only
 the modern good, namely it is producing zero. This result is implausible, since the economy converges to a state with zero
 output, while it can produce much more using raw labor in traditional sectors. A similar problem arises if ρ is zero. In this
 case changes in A have no effect on the equilibrium s , which remains lower than E even if A is very low. This is again an
 implausible result. Hence, both the cases of negative and zero ρ lead to implausible results and that brings us to the
 following restriction.

27 **Restriction 3.** The parameter ρ is positive: $\rho > 0$.

Hence, our analysis of equilibrium leads us to impose some restrictions on the parameters of the CES production
 functions in the model in order to have a well-behaved and stable equilibrium. These required restrictions on the
 parameters are therefore: $\sigma > 0$, $\rho > 0$, and $(\rho - \sigma)/\sigma \leq 0$. We can summarize these three restrictions in the following
 equation:

$$\sigma \geq \rho > 0. \quad (19)$$

31 Since the elasticity of substitution between the traditional and the modern goods is $1/(1-\rho)$, Eq. (19) implies that this
 33 elasticity is higher than 1. Since the elasticity of substitution between intermediate traditional goods is $1/(1-\sigma)$, it is at least
 as high as the elasticity of substitution between the traditional and the modern goods. A similar result holds for the
 elasticity of substitution between modern intermediate goods. We summarize these results in the following proposition:

41 **Proposition 1.** The three restrictions on the parameters ρ and σ , which are required for a stable and well-behaved
 43 equilibrium and which are summarized by Eq. (19), imply:

- a. The elasticity of substitution between modern and traditional goods is higher than 1.
- b. The elasticity of substitution between traditional intermediate goods and between intermediate modern goods is equal
 47 to or even higher than the elasticity of substitution between traditional and modern goods.

49 The elasticity of substitution between the two main goods has significant effects on the dynamics of the economy,
 especially if productivity growth is unbalanced between the two goods. If the elasticity of substitution is high, this sector
 will attract more inputs, while if the elasticity is low and the two goods are complements, the other sector will attract more
 inputs and grow. As described in the introduction, the literature is divided between these two possibilities and this paper
 follows the first line of research. The conclusion of high substitutability between the two main sectors puts this paper in
 contrast especially with [Acemoglu and Guerrieri \(2008\)](#), who reach an opposite conclusion. In their paper one sector is
 capital intensive and the other is labor intensive. Since capital and labor are complementary, it follows that the two goods,
 the capital intensive and the labor intensive, should be complementary as well. In our model the two inputs are raw labor
 and efficiency labor, which substitute rather than complement each other. Hence these differences between the models lead
 to different conclusions on substitutability. Interestingly, the raw labor and efficiency labor of our model are close in spirit to
 low-educated and high-educated labor. According to [Caselli and Coleman \(2006\)](#) the measured elasticity of substitution
 between skilled and unskilled labor is between 1 and 2, where a more precise estimate is 1.4. These numbers fit the results
 of [Proposition 1](#) well.

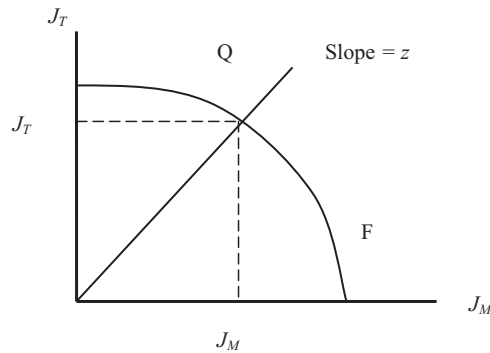


Fig. 2.

4.3. Determination of the numbers of traditional and modern sectors

Fig. 1 describes how the equilibrium ratio between the sizes of the two types of sectors z is determined. We next show how each size, J_T and J_M , is determined. From the labor market clearing conditions (17) and (18) we derive the following relation between the two sizes:

$$\frac{J_T}{L} = \frac{1}{m} \sqrt{1 - \frac{2mJ_M}{E L}}. \quad (20)$$

Note that the square root is always well-defined, since $J_M m$ is total efficiency labor hired, while $LE/2$ is total efficiency labor in the economy, so the ratio between them must be smaller than 1. Eq. (20) describes a negative and concave relation between the number of modern sectors J_M and the number of traditional sectors J_T . This relation, which we call the Sector Frontier, is described by the curve F in Fig. 2. It fits our intuitive description of the increasing marginal opportunity costs of creating new modern sectors in the Introduction. In this model these increasing marginal costs are a result of the heterogeneity of efficiency labor. As more modern sectors are created, more efficiency labor is hired and as a result its average level is reduced, as the higher-efficiency workers are hired first. This increases the cost of creating new modern sectors. Note that the Sector Frontier does not depend at all on the productivity of modern sectors A .

Fig. 2 describes the determination of the equilibrium numbers of traditional and modern sectors in the economy. The curve F describes the relationship between these two variables, while the diagonal line at slope z describes the equilibrium ratio between these two variables, as determined above in Fig. 1. The intersection between the curve F and the line with slope z together determine each of the numbers of the two types of sectors. Once these two numbers are determined, the full equilibrium in the economy is determined with them, through the various equilibrium conditions in Section 3. We can therefore summarize this discussion by the following proposition:

Proposition 2. If χ is non-positive, there is a unique equilibrium, which is described by the numbers of sectors (J_M, J_T) and by the threshold s , and this equilibrium is stable.

5. Economic growth and diversification

In this section we examine the relationship between economic growth and the distribution of sectors in our model. We do it by analyzing how the economy is affected by changes in the productivity of modern sectors A . We first turn to Fig. 1, which describes how the equilibrium is determined. As noted above, changes in A have no effect on the G curve, but as productivity A increases, the M curve shifts down according to Eq. (18) and to our assumption that ρ is positive. As a result, both s and z are reduced. Hence, technical progress reduces the threshold between the two types of labor so that more individuals supply efficiency labor and more people work in the modern sectors. As implied by Fig. 2, the rise in A and the resulting decline in z increase the number of modern sectors and reduce the number of traditional sectors.

This result is strongly related to Proposition 1, according to which the substitutability between traditional and modern goods in the economy is higher than 1. As productivity of modern intermediate goods rises, more workers move to these sectors and their number increases at the expense of traditional sectors. This can happen only if there is high substitutability between the two goods. When production of the modern good increases, more sectors are created and the producers of the modern good use more intermediate goods. This result is related also to high substitutability between these intermediate goods, since otherwise the shift of workers to modern production might increase the size of each sector, but not the number of modern sectors. Similarly this result builds also on reduction of traditional sectors as the economy grows and is thus related to high substitutability between traditional intermediate goods.

We next turn to examine how a rise in productivity A affects not only the numbers of each type of sectors, but also the overall concentration or diversification to sectors in the economy. One measure of such diversification is the total number of

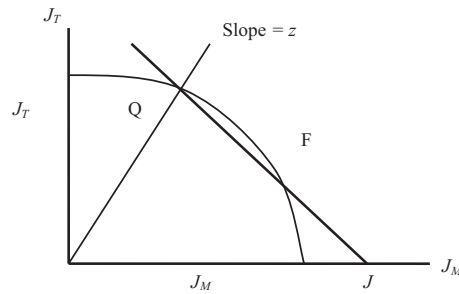


Fig. 3.

sectors, $J = J_T + J_M$. The effect of growth on this diversification can be analyzed diagrammatically, as done in Fig. 3. As in Fig. 2, the intersection of the Sectors Frontier F and the diagonal at slope z determine the numbers of the two types of sectors. But Fig. 3 adds the total number of sectors, which is the intersection of the horizontal axis with a line of slope -1 that passes through the equilibrium point Q .

Fig. 3A rise in productivity A reduces z and thus increases J_M and reduces J_T , since it shifts the economy to the right and down on the curve F . That increases the total sum of sectors J since it shifts the line with slope -1 to the right. As Fig. 3 shows, at some point this result switches and a further shift down the F curve starts to move this line to the left and therefore reduces the total sum of sectors. This switching occurs exactly when the slope of F is equal to -1 , namely when the marginal opportunity cost of building another modern sector is exactly equal to 1. This behavior fits the empirical findings of Imbs and Wacziarg (2003) that sector diversification rises and then declines along the path of economic growth. But this result might not always be obtained in our model, as it requires that the switching point should be inside the Sector Frontier F . This issue is examined in the next proposition.

Proposition 3. The sector frontier F has a point with a slope of -1 between $J_M=0$ and $J_T=0$, if the maximum efficiency E satisfies: $E > 1$. Hence, if $E > 1$, diversification, if measured by the total number of sectors J , follows an inverse U shaped path along economic growth, as it first rises and then declines.

Proof: In the Appendix.

The explanation to this result is the following. The amount of employment in each traditional sector is m . The amount of efficiency labor in each modern sector is the same, but that is not the amount of people employed. Since average efficiency in modern sectors is $(E+s)/2$, the number of employed people in such a sector is

$$\frac{2m}{E+s}$$

When modern sectors begin to form, s is high and close to E , so the number of employees in the first modern sector is actually equal to m/E . Hence, the condition $E > 1$ means that the required number of employees in a modern sector at the beginning of economic growth should be smaller than the required amount in a traditional sector. This is of course in general the condition for the curve F to have an internal point with a slope -1 .

This is therefore the main result of the paper. It is interesting in itself, as most research on this issue focused on monotonic relations between growth and diversification, while this model raises the possibility of a non-monotonic relation. Furthermore, this result is obtained from a very basic assumption in Economics, which is increasing marginal costs. This result is important also because it fits the empirical findings of Imbs and Wacziarg (2003). To fit the model better to these findings we measure in Proposition 4 diversification by the distribution of labor, as done by Imbs and Wacziarg (2003). We actually measure concentration of labor across sectors, by the Herfindahl index, or the Herfindahl–Hirschman Index (HHI), which is an inverse measure to diversification. Proposition 4 examines how concentration is related to economic growth and shows that the conditions for a U shaped relationship are very similar to those in Proposition 3¹⁴.

Proposition 4. If $4 > E > 1$, concentration, as measured by HHI of labor shares in sectors, declines as A begins to grow and then rises from some A on.

Proof: In the Appendix.

Clearly, the lower bound on E in Proposition 4 is the same as the lower bound in Proposition 3, and the intuition behind this result is the same as well. But Proposition 4 imposes also an upper bound on E , which is not imposed in Proposition 3. The reason for that is the following. As shown above, the number of employees in a modern sector is equal to $2m/(E+s)$. It is inversely related to average efficiency, which is correlated with E . As a result, if E is very high, the number of employees in each modern sector is very low. The number of modern sectors also rises with E , but since the size of labor in each sector in

¹⁴ The results are similar if we use other measures, like the Gini coefficient.

the calculation of the HHI is squared, the overall value of HHI becomes very low if E is very high. Hence, above some level of E , the measure of concentration HHI might only decline and not rise. Note also that some upper bound on E is reasonable, since if it is not bounded, there will be too many modern sectors, as each one is very small. This is not a realistic result.

6. Two extensions of the model

This section explores two extensions of the model. The first one assumes that acquiring expertise is costly, but these costs decline with the size of the sector, as firms learn from one another. Hence, this extension enables entry of more firms to each sector and to competition instead of monopoly. The second extension assumes that not everyone can supply efficiency labor, but only people who acquired some human capital. Thus, this extension studies how changes in human capital might affect the equilibrium distribution of sectors.

6.1. Sectors with many firms

In the benchmark model every sector consists of only one firm. The reason is that every new firm prefers to create a new sector, where the demand price is high, than to join an existing sector and face a lower demand price. Note that monopolistic competition in this model is not a result of some fixed costs, but a result of incentives for product differentiation. In order to have more firms in each sector we need to introduce a force that counters the incentives to differentiate. We introduce such a mechanism in this sub-section. It is sketched briefly and the full analysis is left to future research.

Assume that the model is the same as the benchmark model, but that sector expertise does not come for free and learning it requires effort. Assume further that this effort is experienced only in a new sector, while it disappears in an existing sector, where experts learn from others and also from previous producers. In other words, we assume that costs are decreasing with the size of the sector. As a result of this assumption, firms prefer to enter an existing sector, have lower revenues than in a new sector, but have also lower costs of learning the required know-how. The specific assumption we make is that while utility of consumers is equal to c , utility of experts in a new sector is equal to c/f , where $f > 1$ is their effort of learning the required know-how. Clearly, the income required for expertise in a new sector is fw_l in traditional sectors and fw_h in modern sectors.

We next turn to analyze the equilibrium under this assumption, beginning with traditional sectors, but the analysis for modern sectors is similar. Due to symmetry all traditional sectors are the same, $j = 1, \dots, J_T$. The main equilibrium condition is that profits of a competitive firm in each of these sectors should be equal to profits of the first firm in the next potential sector $J_T + 1$, which is a monopoly, and both should be equal to 0. It can be shown that the equilibrium labor input in a competitive firm, for example in the last sector, is:

$$l_{T,J_T} = \frac{\alpha X}{1 - \alpha}.$$

As a result, the zero profit condition for such a firm can be written as:

$$(1 - \alpha)Y^{1-\rho}Y_T^{\rho-\sigma}n^{\sigma-1}l_{T,J_T}^{\alpha\sigma} = w_l X,$$

where n is the number of firms in each traditional sector. Similarly, labor input in a single monopolistic firm in the next new sector is:

$$l_{T,J_T+1} = fX \frac{\alpha\sigma}{1 - \alpha\sigma}.$$

Thus, the zero profit condition for such a firm in the new sector can be written as:

$$(1 - \alpha\sigma)Y^{1-\rho}Y_T^{\rho-\sigma}l_{T,J_T+1}^{\alpha\sigma} = w_l fX.$$

By use of these four conditions we can derive the number of firms in a competitive sector:

$$n = f^{\frac{1-\alpha\sigma}{1-\alpha}} \left(\frac{1-\alpha}{1-\alpha\sigma} \right)^{\frac{1-\alpha\sigma}{1-\alpha}} \left(\frac{1}{\sigma} \right)^{\frac{\alpha\sigma}{1-\alpha}}. \quad (21)$$

Note that this number can be quite high if the effort of starting a new sector f is sufficiently high and as a result there can be competition instead of monopoly in each sector. The number of firms in each modern sector can be shown to be the same as (21). Since the number of firms in each sector is constant across sectors and the number of employees is also constant across sectors and they do not depend on A , all the results of the benchmark model hold in this extension as well. Hence, the main results of the paper do not depend on whether each sector is competitive or monopolistic.

6.2. Cross-country differences in human capital

In this sub-section we extend the model so that it can account not only for differences in productivity A across countries, but also for differences in human capital across countries. Assume that not all workers have efficiency labor, but only those who study and acquire human capital. The number of such people is denoted by H ¹⁵. The rest supply only raw labor and can work only in traditional sectors. We next derive the equilibrium conditions of this extension. The efficiency labor market clearing condition is:

$$J_M m = \int_s^E \frac{H}{E} de = \frac{HE^2 - s^2}{2}. \quad (22)$$

The raw labor market clearing condition is:

$$J_T m = L - H + \int_0^s \frac{H}{E} de = L - H + \frac{H}{E}s. \quad (23)$$

Dividing these two conditions we get that in this extension the G curve is described by:

$$\frac{J_T}{J_M} = 2 \frac{EL/H - (E - s)}{E^2 - s^2}. \quad (24)$$

The M curve in this extension is the same as in the benchmark model and is described by (18) as well. Finally note that the Sector Frontier in this extension of the model is described by the following equation, which is derived from (22) to (23):

$$J_T = \frac{1}{m} \left[L - H + H \sqrt{1 - \frac{2mJ_M}{E H}} \right]. \quad (25)$$

Note first, that in this case the number of traditional sectors is bounded from below, since even if the economy reaches maximum production of modern sectors, $L - H$ workers still work in traditional sectors. The maximum amount of modern sectors is $(H/m)(E/2)$. When human capital increases, the Sector Frontier shifts outward, but not in parallel, as the maximum number of modern sectors increases while the maximum number of traditional sectors remains unchanged, at L/m .

We next turn to examine the effect of a rise in human capital H on the economy. First, according to Eq. (24) such an increase in human capital reduces L/H , so that the G curve shifts to the left. As a result s increases and the ratio of numbers of sectors z declines. This is an interesting result in itself. If the number of people who can supply efficiency labor increases, a smaller share of them goes to modern sectors. It means that the increased supply of efficiency labor does not end up fully in modern sectors. As a result, the average ability of efficiency labor, $(E + s)/2$, increases. As for the effect of a rise in human capital on the numbers of sectors, note that in addition to the decline of the ratio z , the Sector Frontier itself shifts outward. This clearly increases the amount of modern sectors J_M . The effect on the number of traditional sectors J_T is less clear. Next we examine the effect of an increase in H on the total number of sectors J . The derivative of J with respect to H is equal to:

$$\frac{dJ}{dH} = \frac{(E - 1)^2 - (s - 1)^2}{2mE} + \frac{H}{mE}(1 - s) \frac{ds}{dH} \quad (26)$$

The first element in the derivative is always non negative. The derivative ds/dH is non-negative, but $1 - s$ can be negative. We next show that the effect of increase of human capital on the total number of sectors depends on whether the economy is developed or not. Consider first the case of a less developed economy, due to a low A (and could be low H as well). In this case s is very close to E , which makes the first element in the derivative close to zero. But in such equilibrium the M curve intersects G very far to its right, where it is very flat. Hence, the derivative ds/dH is very low as well. Hence, in this case the derivative (26) is close to zero. If on the contrary the economy is developed and s is low and is even below 1, the derivative (26) is positive. We therefore conclude: If the economy is not developed, an increase in human capital does not have a significant effect on the total number of sectors. If the economy is developed, an increase in human capital increases the total number of sectors.

This implies that the effect of an increase in human capital on sector diversification is different to some extent than the effect of an increase in productivity A in our benchmark model. This is not surprising. A rise in productivity A increases the demand for efficiency labor, while a larger H increases its supply. Hence, the effect of economic growth on sector diversification might be somewhat different if we study a model with both technical change and accumulation of human capital. Such a model is outside the scope of this paper, but it is important to keep in mind that over the last two hundred years, most economic growth has been driven by technical change, rather than by human capital, as noted in the introduction.

¹⁵ For simplicity assume that among those who acquire human capital the distribution of efficiency e is the same as in the benchmark model. Thus, this distribution is not affected by changes in H .

7. Conclusion

This paper presents a theory of how sectors are endogenously created and what happens to their distribution along the path of economic growth. The main result of the paper is that, as the economy begins to develop, production become less concentrated and economic activity spreads across a larger variety of sectors. But there is a level of development beyond which production begins to concentrate again over a smaller variety of sectors. In other words, the total number of sectors follows an inverse U shaped curve, which is consistent with the empirical findings of [Imbs and Wacziarg \(2003\)](#).

The paper builds on a very basic assumption in economics, which is increasing marginal costs. We assume that individuals are equally endowed with raw labor, while efficiency labor is randomly assigned across them. This gives rise to increasing marginal opportunity cost of setting up new modern sectors. The reason is that as labor moves from traditional to modern sectors along the growth process, individuals with higher efficiency labor move first and those with lower efficiency move only later. Thus, setting up modern sectors requires fewer workers first, and more workers in later stages of growth. As a result, the marginal reduction of traditional sectors increases. This is the main intuition behind our central result. As modern sectors are created and traditional sectors are eliminated, the net change in the number of sectors for each new modern sector is one minus the alternative marginal cost in terms of traditional sectors. As this marginal cost rises, the net change in the total number of sectors declines, being first positive and then negative.

In addition to the result on the dynamics of diversification our paper also finds some restrictions that are required to guarantee the stability of equilibrium and to make it well-behaved. We show that these restrictions imply that the elasticity of substitution between the modern and traditional goods should be greater than 1, and that the elasticity of substitution between traditional goods themselves and between modern goods themselves should be even higher than that. These results are interesting and seem to be in line with the empirical finding that the elasticity of substitution between high-skilled and low-skilled labor is around 1.4. This is related, since we assume that traditional sectors employ raw labor, while modern sectors employ efficiency labor, which is more human capital intensive.

This is a theoretical paper, but it might have some empirical implications, especially for extending the study of [Imbs and Wacziarg \(2003\)](#) by distinguishing between traditional and modern sectors. According to our model the relation between the numbers of the two types of sectors should form a concave frontier. It also implies that changes in human capital might shift this frontier, while technical change moves the economy along the frontier. Such an empirical analysis can benefit from the recent contribution of [Caselli and Coleman \(2006\)](#), which measures for each country the productivity of high-skilled and of low-skilled labor and shows that these two variables are related across countries by a concave frontier. They also show that this frontier shifts over groups of countries. This similarity can help us to conduct such an empirical analysis, which could give further information on sector dynamics.

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Appendix. Discussion of the case of positive χ

Assume that $\chi > 0$. From Eqs. (17) and (18) we can derive the following equilibrium condition:

$$s = \left(\frac{2s}{E^2 - s^2} \right)^\chi A^{-\rho}. \quad (\text{A.1})$$

The equilibrium s is equal to the ratio of real wages, w_l/w_h , which is the right hand side of (A.1). Hence, if s is above the wage ratio, s declines, while if it is below the wage ratio, s rises. To analyze Eq. (A.1) diagrammatically, we rewrite it slightly differently:

$$sA^\rho = \left(\frac{2s}{E^2 - s^2} \right)^\chi. \quad (\text{A.2})$$

As discussed above, as long as the LHS of (A.2) is above the RHS, s declines, and if the LHS is below the RHS, s rises.

Note, that the LHS of (A.2) is a linear line from the origin with a slope A^ρ . As for the RHS, distinguish between two cases, $\chi < 1$, and $\chi \geq 1$. The RHS is an increasing function of s . As s approaches E , the RHS of (A.2) approaches infinity. The derivative of the RHS of (A.2) with respect to s is:

$$2\chi \left(\frac{2s}{E^2 - s^2} \right)^{\chi-1} \frac{E^2 + s^2}{(E^2 - s^2)^2}.$$

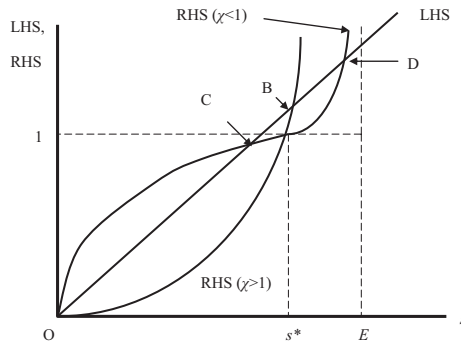


Fig. A1.

Hence, if $\chi \geq 1$, the RHS is convex. If $\chi < 1$, the RHS is first concave, as its derivative at $s=0$ is infinity, and then it becomes convex at some point, since it should rise to infinity as s approaches E . The following diagram presents the LHS and the RHS in the two cases and their intersections. Note that all RHS curves reach the value 1 at the same value of s , $s^* = \sqrt{1+E^2} - 1$. All the curves are presented in Fig. A1.

If χ is greater than or equal to 1, the RHS is convex and there are two points of intersection, O and B . According to the dynamic analysis above, B is unstable. O is a stable equilibrium, but it has a problematic economic implication, namely, that for any level of productivity A , even if it is very low, s is equal to zero so that all workers supply efficiency labor and there are only modern sectors. We therefore should rule out this case.

If χ is smaller than 1, there are three equilibrium points, D , C and O . D and O are unstable. The only stable equilibrium is C . Consider next the effect of reducing productivity A . It shifts the LHS curve down. The reason is that if χ is positive and if we also assume that σ is positive then ρ should be positive as well. As A is reduced, the LHS reaches a point where it is completely below the RHS. In this case, the threshold s jumps up all the way to E , so there is no production in the modern sectors. This means that the equilibrium in this case has a point of non-continuity with respect to productivity A .

Proof of Proposition 3

The slope of the sector frontier, which is actually the marginal cost of setting a new modern sector, is simply the derivative of (20), which is:

$$\frac{\partial J_T}{\partial J_M} = -\frac{1}{E} \left[1 - \frac{2m}{EL} J_M \right]^{-\frac{1}{2}}$$

At $J_M=0$, the derivative is equal to:

$$\left. \frac{\partial J_T}{\partial J_M} \right|_{J_M=0} = -\frac{1}{E}$$

This slope is higher than -1 if $E > 1$. At $J_T=0$ the derivative is minus infinity. QED.

Proof of Proposition 4

The number of workers and experts in each traditional sector is m , while the number of workers and experts in each modern sector is $2m/(E+s)$. Hence the Herfindahl–Hirschman index of the economy is

$$HHI = \sum_{j=1}^J s_j^2 = J_T \left(\frac{m}{L} \right)^2 + J_M \left(\frac{2m}{L(E+s)} \right)^2,$$

where s_j is the share of labor in sector j . Substituting the numbers of sectors of each type from Eqs. (15) and (16) we get after some manipulations:

$$HHI = \frac{m}{LE} \left(s + 2 \frac{E-s}{E+s} \right).$$

Hence, the derivative of the index with respect to s is

$$\frac{\partial(HHI)}{\partial s} = \frac{m}{LE} \left[1 - \frac{4E}{(E+s)^2} \right].$$

As economic growth increases and A grows from 0 on, s declines gradually from E to 0. In the beginning of economic growth s is equal to E and thus the derivative is equal to:

$$\frac{\partial(\text{HHI})}{\partial s} \Big|_{s=E} = \frac{m}{LE} \left[1 - \frac{1}{E} \right].$$

Hence, if $E > 1$ the derivative is positive, HHI rises with s and falls with A . Namely, at the beginning of economic growth sector concentration declines. When A is very high s is close to 0, where the derivative is:

$$\frac{\partial(\text{HHI})}{\partial s} \Big|_{s=0} = \frac{m}{LE} \left[1 - \frac{4}{E} \right].$$

Hence, if $E < 4$ the derivative is negative and concentration declines with s , which means that it rises with A . Hence from some point on, the concentration of sectors rises with economic growth. QED.

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