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Fiscal policy and the real exchange rate under risk

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This paper examines the effect of fiscal expansions on the real exchange rate in a small open economy, where home and foreign assets are imperfect substitutes. There is an hypothesis, raised by Sachs and Wyplosz (1984), Dornbusch and Fischer (1986), and others, that risk and imperfect substitutability of assets can explain why fiscal expansions sometimes create real depreciations, unlike the standard Mundell–Fleming result. This paper examines the issue within an optimizing framework, where risk and imperfect capital mobility are explicitly modeled. The paper comes up with the conclusion that even when assets are imperfect substitutes, and a fiscal expansion crowds out investment and consumption, this crowding out is not full and the real exchange rate always appreciates.

When international capital movements began to play an increasing role in the world economy, greater attention has been given to the analysis of open economies under capital mobility. One of the major results of most studies of small open economies has been that fiscal expansions lead to short-run real appreciations and current account deficits, and to a greater foreign debt in the long run.[1] This result appeared in the early studies of Mundell (1963) and Fleming (1962), who analyzed the problem in an *ad hoc* model, but it has been later verified in a variety of optimizing rational expectations models, such as Sachs (1982), Blanchard (1983), Obstfeld and Stockman (1985), Persson (1985), Persson and Svensson (1985), and others.[2] The intuition behind this result—referred to here as the Mundell–Fleming result—is quite simple. If capital is fully mobile home interest rates are fully determined by world interest rates and are therefore not affected by changes in fiscal policy. Hence, a fiscal expansion (temporary) does not crowd out investment and does not fully crowd out consumption, and it therefore increases the current account deficit.

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But economists have long observed that the Mundell–Fleming result, despite being 'conventional wisdom,' is not in full accordance with the empirical experience. They have observed many situations and many countries in which fiscal expansions have been accompanied with real depreciations and current account surpluses. In a recent article Borensztein (1989) reports on a simple test of correlations between fiscal policy and current accounts in 30 countries, where the results 'show an almost even split between countries which show a positive association between fiscal deficits and current account deficits and countries which show a negative association, with some correlations being quite significant' (Borensztein, 1989, p. 54). Table 1 is reprinted from Borensztein (1989).

The discrepancy between the standard analysis and the observed facts calls for an explanation. Penati (1987) and Borensztein (1989) claim that under some specifications of intertemporal utility, or of investment technology, the Mundell–Fleming result can be reversed. But such explanations are problematic when we try to explain differences between countries, as in Table 1, since it is hard to believe that countries like Germany, the Netherlands, the UK or Switzerland differ so much in technology or in tastes from countries like Sweden, the USA, France, and Denmark, for example.

Another explanation for the facts described in Table 1 was suggested by Sachs and Wyplosz (1984) and later was supported by Dornbusch and Fischer (1986) and Dornbusch (1986). Their argument can be summarized as follows: if home bonds and foreign bonds are imperfect substitutes, due to risk and imperfect capital mobility, then a fiscal expansion raises home interest rates and crowds out investment and consumption by more than in the standard Mundell–Fleming model. If the crowding out is big enough the Mundell–Fleming result can be reversed. According to this argument, the differences between countries in Table 1 can be attributed to differences in risk levels among countries.

This paper examines this argument within an optimizing rational expectations model and shows that it does not hold within such a framework.[3] Indeed, there is a crowding out of investment, due to interest rate effects, but it is less than

TABLE 1. Correlations between fiscal deficits and current account deficits.

Large and positive		Near zero		Large and negative	
Denmark	0.76	Austria	0.09	Finland	−0.15
France	0.40	Canada	0.02	Germany	−0.32
Italy	0.17	Ireland	0.08	Japan	−0.12
Sweden	0.60	Pakistan	−0.03	Netherlands	−0.75
United States	0.89			Norway	−0.42
India	0.80			Switzerland	−0.79
Korea	0.40			United Kingdom	−0.60
Malaysia	0.60			Israel	−0.66
Chile	0.60			Morocco	−0.46
Colombia	0.81			Kenya	−0.19
Australia	0.82			Philippines	−0.69
New Zealand	0.71			Brazil	−0.69
Iceland	0.38				

Source: International Monetary Fund, International Financial Statistics.

full; hence a fiscal expansion still leads to a real appreciation. This result does not depend on how risky home bonds are relative to foreign bonds, and it also does not depend on how sensitive investment is to changes in the interest rate. Hence, this result is fairly robust and quite general.

The paper demonstrates this result by use of a general equilibrium overlapping-generations model, where investment is in human capital. Shocks to the demand for the home good make its price random, and thus home bonds, namely bonds denominated in the home good, are imperfect substitutes to foreign bonds. It is further assumed that even though foreign bonds are fully mobile across countries, home bonds are not and are held at home only. This assumption is necessary for fiscal policy to affect home interest rates, and is thus necessary in order to examine the Sachs–Wyplosz argument. As mentioned above another assumption in the model is that individuals invest in human capital. It is this investment which is sensitive to changes in home interest rates and which is crowded out as a result of a fiscal expansion. The model abstracts from consumption crowding out effects and concentrates on investment only, but that does not affect the generality of the results, since there are no restrictions on how responsive investment can be to changes in interest rates. Furthermore, since the argument raised in the paper is fairly general, it applies to crowding out of consumption as well.[4] This model, therefore, has three assumptions, which enable it to examine the Sachs–Wyplosz hypothesis: home and foreign bonds are risky relative to one another and are therefore imperfect substitutes; home bonds are nontraded and hence home interest rates are affected by fiscal policy; and investment is highly responsive to changes in home interest rate, and thus fiscal expansion crowds out investment. Under these assumptions, the model and its dynamics are analyzed, and it is shown that fiscal expansions always create a real appreciation.

The full analysis of such a model is not easy, as this is a stochastic, highly nonlinear model in essence. In the solution of the model extensive use is made of the contraction mapping theorem, which is the nonlinear analogue of the method of undetermined coefficients in linear models. But use of this technique is left totally to the appendix, while within the paper arguments are presented in a highly intuitive way. Notice that the technique used in the paper is of interest in itself, as the contraction mapping theorem is used not only to prove existence of equilibrium functions, but also to analyze comparative dynamics.

The paper is organized as follows. Section I describes the model. Section II derives the first-order and equilibrium conditions while Section III uses these conditions in order to analyze the dynamics of the economy. In Section IV we examine the effect of a fiscal expansion and show that it causes a real appreciation on impact. Section V discusses the results and the generality of the model and Section VI offers some concluding remarks.

I. The model

Consider a small open economy that produces one non-durable good, which we call the home good and denote by H. This good is produced by labor only in fixed proportions. There is an additional good in the economy, the imported foreign good, which is denoted by F. The economy is an overlapping-generations economy, where individuals live for two periods each, work in both periods, but in the first period can invest in human capital in order to increase their productivity in the second period of life. For the sake of simplicity we assume that there is no population growth and hence that the number of individuals in each generation can be normalized to be equal to one.

I.A. Production

Each young individual produces a fixed amount of the home good H_1 in the first period of life. The productivity of this individual in the second period of life H_2, depends on the amount of investment in human capital in the first period of life, I:

$$\langle 1 \rangle \qquad H_2 = f(I),$$

where f is an increasing concave production function. It is assumed that investment in human capital I must be of home goods only.

I.B. Utility

Assume that individuals derive utility from consumption in the second period of life only. This model, therefore, abstracts from intertemporal consumption considerations. It is further assumed that individuals consume the home good only. Hence, the utility of each individual is:

$$\langle 2 \rangle \qquad U = u(c),$$

where c is consumption of the home good in the second period of life, and u is a concave, increasing utility function. We further assume that u has decreasing absolute risk aversion.

I.C. Exports

In addition to the home good H, there is a foreign good F. The paper's main focus is on the relative price of these two goods, namely the real exchange rate. Let us therefore define the real exchange rate as the price of the foreign good in terms of the home good. Let e_t denote this relative price in period t. It is assumed that the home good is purchased not only at home, but abroad as well, and the foreign demand for H depends on the relative price, on the real exchange rate. It is further assumed that the demand for exports is affected by random shocks to tastes abroad. Hence, this demand for exports can be described by:

$$\langle 3 \rangle \qquad X_t = X(e_t, \varepsilon_t),$$

where X_t is the amount demanded abroad in period t, the ε_t are random shocks, and X is a function that satisfies: $X_1 > 0$, $X_2 < 0$. The sequence $\{\varepsilon_t\}$ is an i.i.d. sequence of random variables, each with expectation zero. The realization of ε_t is known to all in period t, while ε_{t+1} is unknown. We further assume that ε_t is bounded, for all t: $|\varepsilon_t| < \varepsilon$, where $0 < \varepsilon < 1$.

Due to these shocks to the demand for the home good, its price becomes a random variable as well. Hence, the real exchange rate is stochastic.

For the sake of simplicity, let us consider a specific form of the function X:

$$X_t = a\frac{e_t}{1+\varepsilon_t}, \tag{4}$$

where $a>0$. Notice that the demand for exports has elasticity one, with respect to the real exchange rate.

I.D. Assets

There are two financial assets in the economy: foreign bonds, in terms of the foreign good, and home bonds, in terms of the home good. The foreign bonds are traded in a perfect international capital market and they pay a fixed rate of interest r^*. The home bond's rate of interest in period t is r_t. Both interest rates are real, but in terms of different goods. As the real exchange rate is random, the two bonds are risky relative to one another and are imperfect substitutes. We further assume that the home bonds are held by home residents only. Hence, the two bonds differ not only in their realized rates of return, but in their *ex-ante* expected rates of return as well. Thus the home interest rate can differ from r^* and is endogenously determined within the economy. Notice that although capital mobility is limited in this economy, with regard to home bonds, there is still a high degree of mobility, as home residents have full access to the foreign bonds market.[5]

I.E. Fiscal policy

In each period t, the government purchases G_t units of the home good and G_t^F units of the foreign good.[6] The government collects an amount T_t of the home good as taxes, and for the sake of simplicity we assume that taxes are paid by the young only. It is assumed that the government can borrow at home only and hence that it can issue home bonds only. In period t, the government has an outstanding debt of size D_t. The temporal budget constraint of the government in period t therefore implies that:

$$G_t^F = \frac{1}{e_t}[T_t + D_t - D_{t-1}(1+r_{t-1}) - G_t]. \tag{5}$$

As for long-run balancing of the budget constraint, we assume that usually, unless otherwise stated, the government follows a stable policy rule where expenditures of the home good, taxes and public debt are held fixed: $G_t = G$, $T_t = T$, and $D_t = D$. Such a policy is made possible if government expenditures on the foreign good adjust themselves according to equation ⟨5⟩.

I.F. Markets and expectations

We assume that the goods and asset markets in the economy are perfectly competitive. It is further assumed that individuals form their future expectations rationally, having full information on the structure of the economy and on the distribution of the shocks $\{\varepsilon_t\}$.

II. The equilibrium equations

In this section we start to analyze the model, both by studying the optimizing decisions of individuals, and by examining the conditions for equilibrium in the two domestic markets, the market for the home good and the market for home bonds.

II.A. Optimization

Consider an individual who is born in period t, earns an amount H_1 in this period, pays a tax of size T_t, invests I_t in human capital, and allocates the rest of her savings between home bonds HB_t and foreign bonds FB_t. First period budget constraint is therefore:

$$\langle 6 \rangle \qquad H_1 - T_t = I_t + HB_t + e_t FB_t.$$

Future consumption in the second period of life is described by:

$$\langle 7 \rangle \qquad c_{t+1} = f(I_t) + HB_t(1+r_t) + FB_t(1+r^*)e_{t+1},$$

and hence the optimizing problem faced by the individual in period t, can be presented as:

$$\langle 8 \rangle \qquad \max_{I_t, HB_t} E_t u\left[f(I_t) + HB_t(1+r_t) + (H_1 - T_t - I_t - HB_t)(1+r*)\frac{e_{t+1}}{e_t}\right].$$

Hence, the individual allocates her savings between two investments in terms of the home good—investment in human capital and home bonds—and one investment in terms of the foreign good. Let us present $\langle 8 \rangle$ in a slightly different way. If we define B_t to be gross home lending:

$$\langle 9 \rangle \qquad B_t = I_t + HB_t,$$

then $\langle 8 \rangle$ can be rewritten as:

$$\langle 10 \rangle \qquad \max_{I_t, B_t} E_t u\left[f(I_t) - I_t(1+r_t) + B_t(1+r_t) + (H_1 - T_t - B_t)(1+r*)\frac{e_{t+1}}{e_t}\right].$$

There are two first-order conditions for this maximum. The first is with respect to I_t:

$$\langle 11 \rangle \qquad f'(I_t) = 1 + r_t.$$

Therefore, the optimal amount of investment in human capital depends solely on the rate of interest, and it is inversely related to it. Notice that as there are no a priori restrictions on the curvature of the function f, investment can be very sensitive to changes in interest rate. Condition $\langle 11 \rangle$ can be intuitively understood as follows. Since both home bonds and investment in human capital have returns in home good units they should have the same marginal rate of return. As there is an additional asset in the portfolio, namely foreign bonds, there is an additional first-order condition:

$$\langle 12 \rangle \qquad E_t\left\{ u'\left[f(I_t) - I_t(1+r_t) + B_t(1+r_t) + (H_1 - T_t - B_t)(1+r*)\frac{e_{t+1}}{e_t}\right] \times \left[1 + r_t - (1+r*)\frac{e_{t+1}}{e_t}\right]\right\} = 0.$$

This first-order condition determines the allocation of savings between home and foreign assets.

II.B. Equilibrium conditions

Let us first describe the equilibrium in the market for home bonds, where individuals demand bonds and the government supplies them:

$$\langle 13\rangle \qquad HB_t = D_t.$$

An alternative way to present this equilibrium is by gross home lending, which should equal borrowing by investors in human capital and by government:

$$\langle 14\rangle \qquad B_t = D_t + I_t.$$

Next we turn to the market for the home good, where supply is the sum of production by young and old, and demand consists of consumption by old, of government purchases, of investment in human capital by the young, and of exports. The equilibrium condition in the market for the home good is, therefore, described by

$$\langle 15\rangle \qquad H_t + f(I_{t-1}) = c_t + I_t + G_t + X_t,$$

or, if we apply equations $\langle 3\rangle$, $\langle 6\rangle$, and $\langle 7\rangle$, by:

$$\langle 16\rangle \qquad H_1 = I_t + G_t + D_{t-1}(1 + r_{t-1}) + (H_1 - T_{t-1} - I_{t-1} - D_{t-1})(1 + r*)\frac{e_t}{e_{t-1}} + X(e_t, \varepsilon_t).$$

We now have a set of four equilibrium equations, which fully describe the dynamics of the economy. Equations $\langle 11\rangle$ and $\langle 12\rangle$ reflect the optimizing first-order conditions, and equations $\langle 14\rangle$ and $\langle 16\rangle$ describe the equilibrium in the markets for home bonds and the home good.

We now wish to simplify the dynamic equations and reduce the number of endogenous dynamic variables to two: the real exchange rate e_t and investment I_t. This is possible since I_t and r_t are one-to-one related to each other. Thus we can get from $\langle 11\rangle$, $\langle 12\rangle$, $\langle 14\rangle$, and $\langle 16\rangle$ a set of two dynamic equations, namely the first-order condition,

$$\langle 17\rangle \qquad E_t\left\{u'\left[f(I_t) + D_t f'(I_t) + (H_1 - T_t - I_t - D_t)(1 + r*)\frac{e_{t+1}}{e_t}\right] \times \left[f'(I_t) - (1 + r*)\frac{e_{t+1}}{e_t}\right]\right\} = 0,$$

and the home good market equilibrium condition,

$$\langle 18\rangle \qquad H_1 = I_t + G_t + D_{t-1} f'(I_{t-1}) + (H_1 - T_{t-1} - I_{t-1} - D_{t-1})(1 + r*)\frac{e_t}{e_{t-1}} + X(e_t, \varepsilon_t).$$

In the next section we show how these two dynamic relationships determine the dynamics of e_t and I_t through time.

III. The equilibrium dynamics

The dynamic system described by equations ⟨17⟩ and ⟨18⟩ is highly nonlinear and hence very difficult to analyze by use of a linear approximation. There are many reasons for this difficulty, namely the non-quadratic utility and production functions, and the role of expected utility in consumer decision making. But the main two reasons for the inherent nonlinear structure are first the important role of the ratio e_{t+1}/e_t in the dynamics of the system, as it determines the rate of return on foreign assets, and second the fact that e_t is stochastic and hence does not converge (except on average) to a steady state. Fortunately I have been able to find a simple solution to the system of equations ⟨17⟩ and ⟨18⟩ by use of the contraction mapping theorem, following Lucas (1978). The use of this theorem is not surprising, as it is the nonlinear analogue to the method of undetermined coefficients for linear systems. The solution to the dynamic system is presented here in a fairly intuitive way, while the formal proof, using the contraction mapping theorem, is left to the appendix.

We now turn to describe the dynamics of the economy under a stable policy rule, where government purchases of the home good, taxes and public debt are all kept fixed:

$$\langle 19\rangle \qquad G_t = G, \quad T_t = T \text{ and } D_t = D,$$

for all t.

It can be shown that under such a fixed policy there exists a unique rational expectations solution to the model, and that this solution has a rather simple form. More specifically, we show that the rate of investment I_t depends on one variable only, on the real exchange rate in the same period e_t. In other words, there exists a unique nondecreasing function I such that:

$$\langle 20\rangle \qquad I_t = I(e_t),$$

for all t.

The function I is the rational expectations solution of the model in the following sense: if individuals expect investment (and the interest rate, which is inversely related to it) in period $t+1$ to be described by $I(e_{t+1})$, then the equilibrium investment which prevails in period t is also described by the same function I, and is equal to $I(e_t)$. Let us present this argument in a more concrete way. If individuals expect investment in period $t+1$ to be described by $I(e_{t+1})$, then they can form their expectations on e_{t+1} accordingly, by using equation ⟨18⟩, for period $t+1$. Thus ⟨18⟩ becomes:

$$\langle 21\rangle \quad H_1 = I(e_{t+1}) + G + Df'(I_t) + (H_1 - T - I_t - D)(1 + r*)\frac{e_{t+1}}{e_t} + X(e_{t+1}, \varepsilon_{t+1}).$$

Hence, they view e_{t+1} as dependent on e_t, I_t and ε_{t+1}:

$$\langle 22\rangle \qquad e_{t+1} = e(e_t, I_t, \varepsilon_{t+1}).$$

Now individuals can substitute this function e in equation ⟨17⟩, wherever e_{t+1} appears, and hence there emerges a relationship between e_t and I_t. If this relationship is also equal to the same function I, then I is a rational expectations solution to the model. In the next proposition we show that such a function indeed exists and is unique.

Proposition 1: *If the demand for exports is sufficiently sensitive to changes in the real exchange rate, there exists a unique investment function* I, *as described by* $\langle 20\rangle$, *which is consistent with the equilibrium conditions* $\langle 17\rangle$ *and* $\langle 18\rangle$, *under rational expectations.* I *is a nondecreasing function.*

The proof and the precise conditions for this proposition are given in the appendix.

Once the investment function is established we can easily derive the dynamic structure of the model and show that the real exchange rate follows an autoregressive stochastic process, though not linear. If we plug the amounts of investment in periods t and $t-1$ into the equilibrium condition for the home good market, as given by equation $\langle 18\rangle$, we get:

$$\langle 23\rangle \qquad H_1 = I(e_t) + G + Df'[I(e_{t-1})] + [H_1 - T - I(e_{t-1}) - D](1+r*)\frac{e_t}{e_{t-1}} + X(e_t, \varepsilon_t).$$

This equation defines an equilibrium function E:

$$\langle 24\rangle \qquad e_t = E(e_{t-1}, \varepsilon_t),$$

which fully describes the dynamics of the real exchange rate as an autoregressive stochastic process. It is next shown that E satisfies the conditions that make this stochastic process stable.

Proposition 2: *The function* E *satisfies:* $-1 < E_1 < 1$ and E_2 *is non-negative and bounded.*

The proof is given in the appendix.

The two dynamic functions I and E fully describe the evolution of the real exchange rate and investment through time. These dynamics are dependent, of course, on the specific fiscal policy the government is following, namely on G, T, and D. If one or more of these parameters change, the dynamic functions I and E change as well. In the next section we analyze the effect of a fiscal expansion on the dynamics of the economy, and especially on the real exchange rate. We show that the exact changes in the equilibrium function I play an important role in the description of the dynamics of a fiscal expansion.

IV. Fiscal expansion

We can now finally turn to analyze the effect of fiscal expansions on the economy and specifically examine the Sachs–Wyplosz argument. Let us consider the following temporary fiscal expansion: Until period T fiscal policy is stable at $G = G_0$, $T = T_0$ and $D = D_0$. In period T there is an unanticipated temporary rise of G: $G_T = G_0 + \Delta$, which is financed by debt. Hence:

$$\langle 25\rangle \qquad D_T = D_0 + \Delta.$$

Since the change in G is temporary, we have $G_t = G_0$, for all $t \geqslant T+1$. But debt remains larger everafter:

$$\langle 26\rangle \qquad D_t = D_0 + \Delta, \textit{ for all } t \geqslant T.$$

We assume that taxes remain the same, and thus the increased interest payments of the government are made possible by lower G_t^F.[7]

As the fiscal expansion is financed by debt, and the supply of debt rises in period T, that pushes interest rates up and crowds out investment. This highly intuitive result, can be demonstrated in a more formal way. Let I_0 be the investment equilibrium function, which corresponds to the initial fiscal policy (G_0, T_0, D_0). Let I_1 be the new investment function, which corresponds to the new long-run policy $(G_0, T_0, D_0+\Delta)$. Since the new long-run policy is effective from period $T+1$ in the good market, and from period T on in the assets market, it follows that investment in period T is already described by the new function I_1:

$$\langle 27\rangle \qquad I_T = I_1(\varepsilon_T).$$

Thus crowding out of investment means that the function I_1 obtains lower values than I_0, or in other words: $I_1 \leqslant I_0$. The Sachs–Wyplosz hypothesis can therefore be phrased as follows: if I_1 is much lower than I_0, then the crowding out can be more than full and the real exchange rate depreciates. Let us see this explicitly by writing the equilibrium condition in the home good market in period T:

$$\langle 28\rangle \qquad I_1(e_T)+G+\Delta+D_0 f'(I_{T-1}) + (H_1 - T_0 - I_{T-1} - D_0)(1+r*)\frac{e_T}{e_{T-1}} + X(e_T, \varepsilon_T).$$

Hence, if I_1 is smaller than I_0, by more than Δ, then e_T is higher for any realization of ε_T, or e_T is higher on average, namely the real exchange rate depreciates. What we show next is that this can never happen and I_1 cannot fall that much relative to I_0.

Proposition 3: *The crowding out effect on investment is less than full:* I_1 *may be lower than* I_0, *but,*

$$I_1 > I_o - \Delta.$$

The proof is given in the appendix.

Let us describe the intuition behind Proposition 3, as it is indeed the major result of this paper. If crowding out were more than full, that is if I falls by more than Δ, then since public debt rises by Δ, that means that individuals invest less in the home good denominated assets, as $B = I + D$. But the decline of B contradicts the rise of the home interest rate r, since it should make home assets more attractive.

Applying Proposition 3 to equation $\langle 28\rangle$ leads us therefore to the following conclusion:

Corollary to Proposition 3: A temporary debt financed fiscal expansion appreciates the real exchange rate (on average) on impact, and creates a current account deficit.

Proof: It is clear from Proposition 3 and equation $\langle 28\rangle$ that the fiscal expansion lowers e_t relative to what it would have been if there would have been no change. Hence, assuming that we are initially in the neighborhood of a steady state, the real exchange rate appreciates on average (on average—due to the shock ε_t). As for the current account, let us examine the trade balance TB:

$$TB_T = \frac{X_T}{e_T} - \frac{1}{e_T}[T_0 + D_0 + \Delta - D_0(1 + r_{t-1}) - G_0 - \Delta]$$

$$= \frac{a}{1+\varepsilon_T} - \frac{1}{e_T}[T_0 + D_0 - D_0(1 + r_{T-1}) - G_0],$$

according to equations ⟨3⟩ and ⟨5⟩. Thus the decline of e_T reduces the trade balance TB. Since the interest payments on foreign bonds in period T are yet unaffected by the change in policy, being determined at $T-1$, the current account is reduced as well in period T. Hence, the expansionary fiscal policy creates a real appreciation and increases the trade balance and the current account deficits. Q.E.D.

V. Discussion

We next turn to discuss the results of Section IV, and to examine their generality.

V.A. Fiscal policy

It is easy to verify that the specific example of a fiscal expansion which is analyzed in Section IV is representative, and the results of Proposition 3 and its corollary are quite general. In Section IV we assume that the temporary increase in G is debt financed, and the rise in future interest payments, due to greater debt, is paid by cutting down government expenditures of the foreign good. Consider now a similar fiscal expansion, except that from period $T+1$ on taxes are raised, in order to pay the increase in interest payments. Applying a similar method to that used in Section IV, it can be easily shown that in this case too investment is not fully crowded out and the fiscal expansion creates an appreciation on impact. Furthermore, this result holds not only with regard to debt-financed temporary fiscal expansions, but also with regard to a tax-financed temporary expansion as well. This is shown in Zeira (1988a), where it is also proved that a permanent fiscal expansion in such an economy also leads to a real appreciation.[8] Thus our result is not dependent at all on the type of fiscal expansion, and they all appreciate the real exchange rate and deteriorate the current account deficit.

V.B. Robustness of the results

Notice that the results of this paper, as described in Section IV hold for any distribution of the shocks $\{\varepsilon_t\}$, as we have no restriction on that distribution. Hence, the result that a fiscal expansion causes an appreciation is independent of the degree of risk. Notice also that this result holds for every function f of investment in human capital. Even if investment is very sensitive to changes in interest rate, the crowding out effect is always less than full, due to general equilibrium considerations. Hence, the results of this paper hold for any degree of asset substitutability and they also do not depend on how sensitive investment is to changes in the interest rate. It is therefore indicative of the robustness of these results.

V.C. Capital mobility

The original Sachs–Wyplosz hypothesis has been that fiscal expansions may lead to depreciations if three conditions are satisfied: (a) home and foreign assets are imperfect substitutes, (b) home bonds are not perfectly traded abroad, and (c) domestic demand (consumption or investment) is negatively related to home interest rates. The necessity of conditions (a) and (c) is clear, but it can be shown that condition (b) is necessary for that hypothesis too. Consider an economy with perfect capital mobility of both home and foreign bonds. In such an economy the interest rate is not affected at all by the amount of government debt, since the demand for such bonds is infinitely elastic. Hence, there is no crowding out of investment and a fiscal expansion unambiguously causes a real appreciation. This paper shows that even when capital mobility is imperfect and home bonds are a nontraded asset, a fiscal expansion still leads to a real appreciation on impact. In order to view this result in perspective, let us compare it to the case of no capital mobility at all, where home bonds are nontraded and home residents have no access to the foreign bonds market (capital controls). In that case B_t is always equal to $H_1 - T$, and the equilibrium condition in the market for home bonds is:

$$\langle 29\rangle \qquad H_1 - T = D_t + I_t.$$

Hence, a debt-financed fiscal expansion lowers investment by exactly Δ and there is a full crowding out effect. We can therefore view the case discussed in this paper, of limited capital mobility as an intermediate case between full capital mobility and no mobility at all.

VI. Summary and conclusions

This paper carries some good news and some bad news. The good news is that the basic Mundell–Fleming result, that under capital mobility a fiscal expansion creates a real appreciation and increases the trade deficit, is a very robust result. Even when there is a risk and the home and foreign assets are imperfect substitutes, and even when home bonds are traded at home only, and a fiscal expansion drives home interest rates up and crowds out domestic demand, this crowding out is less than full and the real exchange rate still appreciates. Thus the original Mundell–Fleming result holds even when risk and imperfect capital mobility are explicitly introduced into the model.

The bad news is that we are still short of an explanation why in real life budget deficits are sometimes negatively correlated with trade deficits. This empirical observation cannot, therefore, be accounted for by risk and imperfect capital mobility, as suggested by Sachs and Wyplosz (1984) and Dornbusch and Fischer (1986). We should look for an explanation elsewhere.[9]

Appendix

Proof of Proposition 1

Let us first write down the dynamics of the economy by use of three equations:

⟨A1⟩

$$E_t\left\{u'\left[f(I_t) - I_t f'(I_t) + B_t f'(I_t) + (H_1 - T - B_t)(1 + r*)\frac{e_{t+1}}{e_t}\right] \times \left[f'(I_t) - (1 + r*)\frac{e_{t+1}}{e_t}\right]\right\} = 0;$$

$$\langle A2\rangle \qquad B_t = D + I_t;$$

and

$$\langle A3\rangle \qquad I_{t+1} + G + Df'(I_t) + (H_1 - T - D - I_t)(1+r*)\frac{e_{t+1}}{e_t} + a\frac{e_{t+1}}{1+\varepsilon_{t+1}} = H_1.$$

It can be shown that if a is large enough there exist boundaries: $\underline{I}$, $\bar{I}$, $\underline{e}$, $\bar{e}$, such that:

1. $1 < f'(\bar{I}) \leqslant (1+r*)\underline{e}\cdot(\bar{e})^{-1}$;
2. $f'(\underline{I}) \geqslant (1+r*)\bar{e}(\underline{e})^{-1}$;
3. If $\underline{I} \leqslant I_t, I_{t+1} \leqslant \bar{I}$, and $\underline{e} \leqslant e_t \leqslant \bar{e}$, then e_{t+1} also satisfies: $\underline{e} \leqslant e_{t+1} \leqslant \bar{e}$, according to ⟨A3⟩.

Notice that this existence is guaranteed as $\partial e_{t+1}/\partial e_t < 1$ according to ⟨A3⟩.

We can now turn to prove the existence of the function I. Let V be the following closed set of the Banach space $C[\underline{e}, \bar{e}]$:

$$\langle A4\rangle \qquad V = \{J : J : [\underline{e}, \bar{e}] \to [\underline{I}, \bar{I}],\ J \text{ continuous and nondecreasing}\}.$$

We now define a mapping ϕ, $\phi: V \to V$, and show that it is a contraction. Let $J \in V$. If we substitute J in equation ⟨A3⟩ we get:

$$\langle A5\rangle \qquad J(e_{t+1}) + G + Df'(I_t) + (H_1 - T - D - I_t)(1+r*)\frac{e_{t+1}}{e_t} + a\frac{e_{t+1}}{1+\varepsilon_{t+1}} = H_1.$$

If a is large enough, equation ⟨A5⟩ determines a unique value of e_{t-1} (even if $H_1 - T - D - I_t$ is negative), such that: $\underline{e} \leqslant e_{t+1} \leqslant \bar{e}$. Hence ⟨A5⟩ determines a function $\tilde{J}$:

$$\langle A6\rangle \qquad e_{t+1} = \tilde{J}(e_t, I_t, \varepsilon_{t+1}),$$

where $\tilde{J}_2 > 0$, $\tilde{J}_3 > 0$ and $\tilde{J}_1$ depends on whether the economy is a foreign lender $(H_1 - T - D - I_t > 0)$ or a foreign borrower $(H_1 - T - D - I_t < 0)$. But in both cases e_{t+1}/e_t falls as e_t rises. Hence, we can define another function:

$$\langle A7\rangle \qquad \bar{J}(e_t, I_t, \varepsilon_{t+1}) = \frac{\tilde{J}(e_t, I_t, \varepsilon_{t+1})}{e_t} = \frac{e_{t+1}}{e_t},$$

such that: $\bar{J}_1 < 0$, $\bar{J}_2 > 0$, $\bar{J}_3 > 0$.

We can now substitute $\bar{J}$ in equation ⟨A1⟩ and get an optimal B_t which depends on I_t and e_t, and it can be shown that since u has decreasing risk aversion, B_t depends negatively on I_t and positively on e_t.[10] From the equilibrium condition in the home bonds market ⟨A2⟩ we therefore get:

$$\langle A8\rangle \qquad B_t(I_t, e_t) = D + I_t.$$

This determines a function:

$$\langle A9\rangle \qquad I_t = \hat{J}(e_t),$$

which is nondecreasing and is bounded by $\underline{I}$ and $\bar{I}$. Hence, $\hat{J} \in V$. Let $\hat{J}$ be defined as $\phi(J)$. Thus ϕ is a mapping from V into itself. V is a contraction mapping since:
(1) ϕ is monotone.

Let $J_1 \geqslant J_2$. It is clear from ⟨A5⟩ that $\bar{J}_1 \leqslant \bar{J}_2$ and hence $\bar{J}_1 \leqslant \bar{J}_2$. If the foreign asset is having a lower return in every state of nature, then individuals hold less of it, and hence B_t which corresponds to J_1 is greater than the B_t which corresponds to J_2. Hence: $\hat{J}_1 \geqslant \hat{J}_2$, as seen in ⟨A8⟩. Therefore, $\phi(J_1) \geqslant \phi(J_2)$ and ϕ is monotone.
(2) ϕ is contraction.

Remember that the boundaries satisfy $(1+r*)\underline{e}(\bar{e})^{-1} > 1$. Therefore, there exists $\beta < 1$ such that $\beta(1+r*)\underline{e}(\bar{e})^{-1} = 1$. Let $J \in V$, and $\delta > 0$. We want to show that $\phi(V+\delta) \leqslant \phi(V) + \beta\delta$.

If we substitute $J+\delta$ and $\phi(J)+\beta\delta$ in $\langle A3\rangle$ in the place of I_{t+1} and I_t, accordingly, we get: $e_{t+1}/e_t \geq \bar{J}$. Hence, B_t is smaller, which contradicts a larger I_t. Thus $\phi(J+\delta)$ should be lower than $\phi(J)+\beta\delta$.

Since ϕ satisfies these two conditions, it is a contraction, as shown in Stokey and Lucas (1989). Hence, there exists a unique fixed point I, such that:

$$\langle A10\rangle \qquad \phi(I)=I,$$

and I is the rational expectations solution of the system. Q.E.D.

Proof of Proposition 2

From equation $\langle 23\rangle$ and from our assumptions that a is large enough and $e_t \geq \underline{e}$, it is clear that $E_2 \geq 0$ and is bounded from above.

$E_1 < 1$, since if the contrary holds e_{t+1}/e_t rises as e_t rises, for all values of ε_{t+1}. If e_{t+1}/e_t rises and I_t rises too, then B_t falls, which contradicts the increase in I_t, as seen in equation $\langle 14\rangle$. Hence, $E_1 < 1$.

It is clear from $\langle 23\rangle$ that $E_1 > -1$ if a is sufficiently high. Q.E.D.

Proof of Proposition 3

Let ϕ_1 be the contraction mapping described by Proposition 1, which corresponds to the new long-run fiscal policy $(G_0, T_0, D_0+\Delta)$, while ϕ_0 is the contraction mapping which corresponds to the initial fiscal policy (G_0, T_0, D_0). Let us look at the function J defined by:

$$\langle A11\rangle \qquad J(e)=I_0(e)-\Delta.$$

Our goal is to show that $J < I_1$.

First, we show that $\phi_1 J > J$. Since if we apply both $I_{t+1}=J(e_{t+1})$ and $I_t=J(e_t)$ to equation $\langle A3\rangle$ we get:

$$\langle A12\rangle \qquad I_0(e_{t+1})-\Delta+G_0+(D_0+\Delta)f'[I_0(e_t)-\Delta] + [H_1-T_0-D_0-I_0(e_t)](1+r*)\frac{e_{t+1}}{e_t}+a\frac{e_{t+1}}{1+\varepsilon_{t+1}}=H_1.$$

As $1<f'[I_0(e_t)-\Delta]=1+r$, we deduce from $\langle A12\rangle$ that e_{t+1}/e_t is lower than under I_0 and ϕ_0. Hence, foreign assets have a lower yield. As home assets have a higher yield at a smaller investment, we deduce that B_t is higher. But,

$$\langle A13\rangle \qquad B_t=D_0+\Delta+I_t=D_0+\Delta+I_0(e_t)-\Delta=D_0+I_0(e_t),$$

and this contradicts a higher B_t. Thus I_t must be higher. Hence:

$$\langle A14\rangle \qquad \phi_1 J > J.$$

But ϕ_1 is a monotone operator and thus the sequence $\{\phi_1^t J\}$ is an increasing sequence. Since ϕ_1 is a contraction mapping, we know that this sequence converges to the fixed-point I_1 and hence:

$$I_1=\lim_{t\to\infty}\phi_1^t J > J = I_0-\Delta.$$

Q.E.D.

Notes

1. In principle, there are two issues involved: the effect of policy on the real exchange rate and its effect on the current account and the trade balance. But in this paper, as in most of the existing literature, we view the real exchange rate and the trade balance as moving together, being positively related. Hence, the two issues coincide.

2. In overlapping-generations models all fiscal expansions create current account deficits, as shown in Persson (1985) and others. In infinite-horizon models, such as in Sachs (1982) or Blanchard (1983), this result holds for temporary fiscal expansions only. Anyway, in this paper, we examine temporary fiscal expansions.
3. The original Sachs and Wyplosz (1984) argument is made within an *ad hoc* model, which does not incorporate risk explicitly in the model.
4. Earlier versions of this paper have dealt with the issue of crowding out of consumption. See Zeira (1988a).
5. The assumption that home bonds are not traded abroad can be justified by informational asymmetrics, see Zeira (1987).
6. The government is therefore the sole importer of the foreign good in this economy.
7. The case where the increased interest payments are financed by higher tax rates has similar results.
8. More accurately, it is shown that these results hold for countries which are net foreign lenders or have a relatively small foreign debt.
9. In Zeira (1988b) we examine the possibility that this observation can be explained by credit market imperfections. Other possible explanations for this observation may arise when we consider price stickiness and the tax structure in various countries.
10. The full proof of this result is tedious and thus omitted.

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