Cash-in-advance or delayed deposits implications for inflation and growth

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The paper presents a monetary growth model in which the assets market precedes the goods market in each period instead of following it as in standard cash-in-advance models. As a result of this change in timing, money is held by sellers and not only by buyers, and it is shown that as a result inflation reduces the rate of capital accumulation.

1. Introduction

In a monetary economy, where money serves as a medium of exchange, money must be held by agents for some time even in the existence of assets with higher rates of return, as the transactions between money and goods and between money and assets cannot be simultaneous. One way to model this fact is in the cash-in-advance framework, as developed by Clower (1967), Lucas (1980), Helpman (1981), Stockman (1981), Svensson (1985) and others. But the cash-in-advance models concentrate on money holding by buyers and they overlook what happens to sellers in a monetary economy. This paper shows that the fact that sellers must hold money as well has significant economic implications.

Standard cash-in-advance models assume, in addition to the assumption that money is needed to buy goods, that asset markets operate after the goods market closes, so that buyers have to hold money in advance. In this paper this timing is reversed and the goods market operates after the asset market closes. Hence, sellers are stuck with the money they are paid with until the next period. Thus money is held within this framework, which we call delayed deposits, by firms only. This is of course an abstraction from reality, where money is held by both buyers and sellers, but it helps in concentrating on the effects of money holding by firms. The model is then applied to the issue of inflation and growth and it is shown that inflation has a negative effect on capital accumulation. An intuitive explanation to this result is that if firms are holding money they pay a share of the inflation tax. Hence, a higher rate of inflation raises the amount of inflation tax firms pay and thus lowers profitability and slows down capital accumulation.

Notice for comparison that in standard cash-in-advance models money is super-neutral if the cash constraint holds for consumption goods only, as shown by Stockman (1981) and Abel (1985). In our model we get a different result even with the same cash constraint. Thus money holding by sellers has important economic effects.

The paper is organized as follows. Section 2 presents the delayed deposits framework. Section 3 derives the equilibrium, while section 4 examines the effect of a higher rate of inflation on the economy. Section 5 summarizes the paper.

2. The model

Consider a closed economy, in which a single physical good is produced, consumed and invested. There are two assets in the economy: money and capital. Money is assumed to pay no interest, but is held because consumption goods can be purchased by money only.

There is a representative infinitely-lived consumer in the economy, who inelastically supplies one unit of labor each period and who maximizes life-time utility:

$$\sum_{t=0}^{\infty} (1 + \rho)^{-t} u(c_t),$$ (1)

where c_t is consumption in period t, u is a concave utility function, which satisfies the Inada conditions and ρ is the subjective discount rate.

Firms are producing the single good with labor and capital:

$$y_t = F(k_t, n_t),$$ (2)

where y_t is output in period t, k_t is capital input, n_t is labor input and F is a standard CRS production function, which satisfies the Inada conditions. Capital is invested one period ahead of time and does not depreciate.

Government pays the consumer an amount S_t of money as transfer payment, which is financed by printing new money. The policy rule followed by the government is that of a fixed positive rate of monetary expansion μ:

$$S_t = \mu M_{t-1},$$ (3)

where M_{t-1} is the amount of money in period $t - 1$.

Let us now describe the daily order of events, assuming that a day is our unit of time. The goods market operates daily at noon, while the asset market, which can be thought of as a bank, is open by mornings only. Consumers go by morning to the bank, sell capital and get money for consumption purchases of that day. Firms are paid at noon and are therefore stuck with the money they receive until the following morning.

It is further assumed that all markets are perfectly competitive and that expectations are formed rationally by all agents.

3. Equilibrium

As money pays no interest individuals hold capital only overnight. Hence the evolution of individual wealth a_t is:

$$(1 + r_{t-1})a_{t-1} + w_t + s_t - c_t = a_t,$$ (4)

where w_t is real wage, $s_t = S_t/P_t$ is the real value of the transfer payment and r_{t-1} is the real rate of return on capital purchased at $t-1$. Maximization of individual utility (1), given the dynamic constraint (4), leads to the first-order condition:

$$(1 + \rho)\frac{u'(c_t)}{u'(c_{t+1})} = 1 + r_t, \quad \text{for all } t \geq 0, \tag{5}$$

and to the transversality condition:

$$a_t u'(c_t)(1 + \rho)^{-t} \xrightarrow[t \to \infty]{} 0. \tag{6}$$

Firms profit maximization and the equilibrium in the labor market lead to:

$$w_t = F_n(k_t, 1). \tag{7}$$

The equilibrium in the market for money is reached when the amount of money M_t is equal to the consumers' demand for money:

$$M_t = P_t c_t. \tag{8}$$

Equilibrium in the capital market prevails when private total wealth equals the net value of firms, which is equal to the sum of productive capital and cash holding:

$$a_t = k_{t+1} + M_t/P_t. \tag{9}$$

The equilibrium condition in the goods market is given by:

$$k_{t+1} = k_t + f(k_t) - c_t, \tag{10}$$

where f is the per capita production function: $f(k) = F(k, 1)$.

From eqs. (4), (5), (7), (8), (9) and (10) we get:

$$(1 + \rho)\frac{u'(c_t)}{u'(c_{t+1})}(k_{t+1} + c_t) = k_{t+1} + k_{t+1}f'(k_{t+1}) + \frac{1}{1 + \mu}c_{t+1}. \tag{11}$$

This equation can be interpreted as describing firms' total yields. On the left-hand side we have the value of firms' assets multiplied by the gross rate of return, while on the right-hand side is the sum of non-depreciated capital, k, production profits $kf'(k)$ and discounted value of money M_t.

The model is now fully summarized by the two dynamic equations (10) and (11) and by the transversality condition (6). It can be shown that under these conditions the economy converges on a stable saddle path to the steady state [1]. The steady state of the economy is a pair (k, c) which satisfies:

$$k_{t+1} = k_t = k \quad \text{and} \quad c_{t+1} = c_t = c, \tag{12}$$

for all t and for which the equilibrium conditions hold as well.

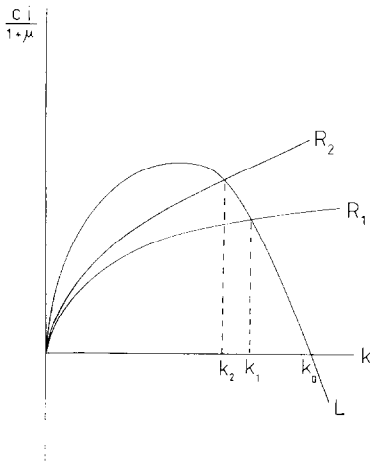
[1] A precise proof of this result appears in Zeira (1988).

Fig. 1

It is easy to verify that in the steady state we have:

$$[f'(k) - \rho]k = \left(\rho + \frac{\mu}{1+\mu}\right)f(k),$$

(13)

which is a condition that determines the steady state. This is shown in fig. 1, where the L curve is the L.H.S. of (13) and the R curve is the R.H.S. of (13). Notice that at the steady state the marginal productivity of capital exceeds the subjective rate of time preference.

4. The effect of inflation

We can now analyze the effects of changes in the rate of inflation, or in the rate of monetary expansion, on the economy. Let us first consider the effect of a rise in μ on the steady state amount of capital k. As shown in fig. 1 this rise shifts the R curve upward and hence the steady state amount of capital k falls. [2] Thus a higher rate of inflation leads to a decline of capital in the long run. [3]

5. Summary

This paper claims that the original cash-in-advance framework should be extended to deal with money holdings by sellers, as well as by buyers. Such an extension can significantly affect the economic analysis, as shown in this paper with respect to inflation and growth, and it is also more realistic as money is held in real life by both firms and individuals.

[2] The dynamics of convergence to the new steady state are fully described in Zeira (1988).

[3] Notice that as inflation is reduced to $-\rho/(1+\rho)$ the amount of capital becomes k_0, and this is indeed Friedman's (1969) optimal rate of monetary inflation. Here this optimal rate gains a new meaning: it is the unique rate which sets the steady state amount of capital at the optimal non-monetary level.

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