Journal of Economic Dynamics and Control 14 (1990) 53–63. North-Holland

COST UNCERTAINTY AND THE RATE OF INVESTMENT

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Received February 1989

This paper examines the effect of cost and price uncertainty on the optimal rate of investment by firms within a general equilibrium framework. Former works which have studied this issue like Hartman (1972) and Abel (1983, 1984, 1985) have come to the conclusion that this effect is positive, since increased price uncertainty raises expected profitability. But these studies assume a fixed discount rate, which is equivalent to assuming that consumers are risk-neutral. This paper shows that if consumers are risk-averse the effect of uncertainty may be reversed and investment may be depressed when price or cost uncertainty is increased.

1. Introduction

This paper examines the effect of price and cost uncertainty on the rate of investment of competitive firms. This theoretical issue has been analyzed by Hartman (1972), Pindyck (1982), and Abel (1983, 1984, 1985) and the final conclusion of this series of papers is that an increase in such uncertainty raises the rate of investment. The reason for this positive effect is that since optimal profits are a convex function of relative prices, price and cost uncertainty raise expected marginal profits and increase capital accumulation. These papers did not take into consideration another channel through which uncertainty affects investment, which is through shareholders' risk aversion.[1] Through this effect uncertainty tends to reduce consumers' demand for capital and so to reduce investment.[2] The two effects therefore work in opposite directions. This paper presents a general equilibrium model that explicitly incorporates these two channels of influence and takes both effects into consideration.

Risk aversion of shareholders is left out in the above mentioned papers by assuming that the rate of discount of firms is fixed and unaffected by changes in uncertainty. This occurs only in two cases: if shareholders are risk-neutral

*I wish to thank Andrew Abel, Roger Craine, Jeffrey Sachs, and two anonymous referees for valuable comments. Remaining errors are all mine.

[1]Abel (1985) acknowledges the possibility of this effect but does not incorporate it in the model.

[2]Zeira (1987, 1988) presents two examples where increasing risk reduces capital accumulation in general equilibrium models.

0165-1889/90/$3.50

or if the rate of return is uncorrelated with the return on market portfolio. The two returns can be uncorrelated through changes in risk only if the sector we analyze is negligible relative to the market portfolio. Each of these two assumptions, of risk neutrality or of negligibility, severely restricts the application of these models and their economic importance. Hence constructing a model which takes into consideration that firm owners are risk-averse seems to be a necessary development in the line of research which has been initiated by Hartman (1972).[3]

This paper therefore examines optimal capital accumulation of value-maximizing firms, as in Hartman and Abel, but this value is determined in the capital markets by risk-averse participants. In the model developed here price uncertainty raises expected profitability on the one hand but increases capital risk on the other hand, and hence the effect on capital accumulation is ambiguous. It is hard to draw the precise lines where one effect is stronger than the other, even in the simple model used in this paper. Nevertheless the paper demonstrates through various examples that the answer to this question depends on many aspects: degree of risk aversion, convexity of the profit function, and the distribution of risk.

The model developed in the paper is an as simple as possible overlapping generations model, where consumers diversify their portfolio between two investments of which one is with variable random costs. The alternative asset is riskless but the analysis can be extended to a risky asset as well. Throughout the discussion capital is assumed to be costlessly adjusted, but the main results hold in the case of adjustment costs to investment as well.[4] Looking at cost uncertainty of course fully accounts for price uncertainty as well, since it is the variability of the relative price of inputs to outputs that matters.

The paper is organized as follows. Section 2 presents the basic model for which section 3 derives the basic equilibrium conditions. Section 4 discusses a simple example and section 5 analyzes the effect of risk aversion and of the profit function on the results. Section 6 summarizes the paper.

2. The model

Consider an economy with one physical good, which is used both for consumption and for investment. This good can be produced in three different technologies; technology I uses labor as the only input, technology II uses capital only, and in technology III the good is produced by labor and capital together.

[3] Indeed, Craine (1989) has conducted independently and virtually simultaneously a research along this line. The two papers, though, are quite different.

[4] Analysis of this model in case of adjustment costs can be found in Zeira (1989).

Production by labor according to technology I is assumed to be linear and is described by

$$Y_t^L = W_t \cdot L_t^L, \tag{1}$$

where Y_t^L is output and L_t^L is labor input in this technology. The marginal productivity of labor W_t changes from period to period due to random technological shocks. We assume that W_t is a random variable which is realized only at time t, and that $\{W_t\}_t$ is an i.i.d. series of random variables. The average marginal productivity of labor is $W_0 = \mathrm{E}_{t-1}(W_t)$ for all t. It is further assumed, for purposes of tractability, that W_t is bounded: $0 < \underline{W} < W_t < \overline{W} < \infty$. The random W_t plays the role of uncertain costs in this model.

Production by capital only according to technology II requires investment one period ahead of time, creates no depreciation, and is assumed to be linear:

$$Y_t^K = r_0 \cdot K_t^K, \tag{2}$$

where Y_t^K is net output and K_t^K is capital input in this technology. The marginal productivity of capital in this technology r_0 is positive. It thus serves as a riskless rate of return.

The third technology requires installation of capital one period ahead of time, involves no depreciation, and is described by

$$Y_t = F(K_t, L_t), \tag{3}$$

where Y_t is net output in technology III and K_t and L_t are capital and labor inputs in this technology. F is a standard concave production function with constant returns to scale. It is further assumed that there are no adjustment costs to investment and hence capital is instantaneously adjusted. This assumption is not limiting since it does lead to the same qualitative results as in the costs of adjustment case.[5]

Consumers in this model live for two periods each in overlapping generations: work and save in the first period of life and consume in the second. Individuals are assumed not to consume in first period of life in order to abstract from the precautionary effect of uncertainty on saving and to concentrate on the portfolio effects only. The young supply one unit of labor in the first period of life and derive utility from consumption when old.

[5]Hence the results of the paper are comparable to those of Hartman and Abel, even if they use a gradual investment model.

Utility from consumption in period t is

$$u(C_t) = \frac{1}{1-\alpha} C_t^{1-\alpha}, \qquad \alpha > 0, \tag{4}$$

or

$$u(C_t) = \log C_t, \tag{5}$$

where C_t is consumption in period t by the old. Utility is assumed to be of constant relative risk aversion, which is α in (4) or 1 in (5). The young in period t therefore maximize $\mathrm{E}_t\, u(C_{t+1})$.

It is assumed that there is no population growth in the economy and hence the size of each generation can be normalized to one.

An additional assumption on the technologies in this economy is

$$F_L(\overline{W}, 1) < \underline{W}. \tag{6}$$

This assumption guarantees the existence of a pure labor production sector of technology I in all situations. Some labor must always be employed in sector I since otherwise wages in the technology III sector exceed W_t and that is impossible even when capital in this sector is maximal, as shown by condition (6).

It is further assumed that all markets are perfectly competitive and that expectations are rational. Thus consumers and firms have full information on both the model described above and of the distribution of productivity shocks.

3. Equilibrium

Since production in technology I by labor only is always positive, due to condition (6), the labor market equilibrium real wage must equal W_t in each period t. Hence firms face a random real wage which is unknown when investment decisions are made. Firms' profit maximization in period t, when the amount of capital K_t has already been determined in $t-1$, is described by

$$\max_{L_t} \left[F(K_t, L_t) - W_t L_t \right] = K_t \Pi(W_t). \tag{7}$$

The function Π, which is defined by eq. (7), is decreasing and convex. Notice that it is the convexity of Π which accounts for the results of Hartman and Abel, namely for the positive effect of uncertainty on investment.

The young work one unit of time, earn W_t units of the good, and save it all. The savings are allocated between K_t^K and K_t, between the pure capital technology and the capital–labor technology. Let x_t denote the share of saving that is invested in technology III, of production with both capital and labor.

Utility maximization of the young is therefore described by

$$\max_{0 \leq x_t \leq 1} \mathrm{E}_t u\{W_t x_t[1 + \Pi(W_{t+1})] + W_t(1 - x_t)(1 + r_0)\}. \tag{8}$$

Since the utility function is of constant relative risk aversion, as described by eqs. (4) and (5), maximization of expected utility is equivalent to the following optimization:

$$\max_{0 \leq x \leq 1} \mathrm{E}_t u\{x[1 + \Pi(W_{t+1})] + (1 - x)(1 + r_0)\}. \tag{9}$$

Notice that x_t is time-independent since it no longer depends on W_t. Hence x_t is denoted by x. If optimal portfolio allocation is not a corner solution of (9) it is presented by the following first-order condition:

$$MU = \mathrm{E}_t u'\{x[\Pi(W_{t+1}) - r_0] + 1 + r_0\}[\Pi(W_{t+1}) - r_0] = 0. \tag{10}$$

Since $MU_x < 0$, the second-order condition is always satisfied.

The equilibrium in the capital market determines the allocation of savings between capital in the pure capital sector and in the labor–capital sector. Capital in this sector, which uses technology III, is equal to

$$K_{t+1} = W_t x, \tag{11}$$

where x is determined in portfolio optimization described by (9). Capital therefore fluctuates due to productivity shocks in technology I, that cause fluctuations in savings, but the average amount of capital is determined by x. The share x depends on the distribution of W, on r_0, on the production function F, and on the utility function u. The issue this paper deals with is how an increase in the uncertainty of the future cost variable W_{t+1} affects capital accumulation or the average amount of capital K_{t+1} in the technology III sector. Hence the paper focuses on the effect of increased uncertainty of W on the share x. Notice that the analysis of the effect of changes in the distribution of W_{t+1} cannot lead to clear-cut results since utility depends on W_{t+1} through a concave function u imposed on a convex function Π. I've found it hard to arrive at an overall specification as to when increased uncertainty leads to more capital and when to less. But the paper presents enough results that point out how this effect depends on the various characteristics of the model. One main result though emerges clearly from the following sections: if consumers are risk-averse, increasing cost and price uncertainty can either raise investment or decrease it.[6]

[6] The reaction of investment to uncertainty under risk aversion depends, as Abel (1985) claims, on the covariance of returns in the specific sector and in the whole market. But since in our model this covariance depends on the share of the sector in the economy, which itself depends on capital accumulation, it is necessary to look for more fundamental determinants of investment behavior.

4. Profitability and the alternative rate of return

In this section we begin to examine the effect of an increase in cost uncertainty on x through a specific distribution in which probability is concentrated in the two symmetric boundary points. We call this distribution the two-point distribution and it is described by

$$W_t = \begin{cases} W_0 + \varepsilon & \text{in probability } \tfrac{1}{2}, \\ W_0 - \varepsilon & \text{in probability } \tfrac{1}{2}, \end{cases} \tag{12}$$

where ε is a positive and relatively small number. Searching for the effect of increased uncertainty on capital accumulation in this case means looking at the effect of changes in ε on x. In order to simplify the analysis even more let us assume that utility is the logarithmic function (5). In this specific case the optimal share x is given by

$$x = \frac{1+r_0}{2}\left[\frac{1}{r_0 - \Pi(W_0+\varepsilon)} - \frac{1}{\Pi(W_0-\varepsilon) - r_0}\right], \tag{13}$$

if $\Pi(W_0-\varepsilon) > r_0$, $\Pi(W_0+\varepsilon) < r_0$, and x is not a corner solution.

Even in this specific example the answer to our basic question, what if the sign of $\mathrm{d}x/\mathrm{d}\varepsilon$ is ambiguous, as can be seen by presenting the share x as a difference: $x = x_1 - x_2$, where x_1 and x_2 are given by

$$\begin{aligned} x_1 &= \frac{1+r_0}{2}\,\frac{1}{r_0 - \Pi(W_0+\varepsilon)}, \\ x_2 &= \frac{1+r_0}{2}\,\frac{1}{\Pi(W_0-\varepsilon) - r_0}. \end{aligned} \tag{14}$$

Notice that both x_1 and x_2 are decreasing functions of ε and hence the sign of $\mathrm{d}x/\mathrm{d}\varepsilon$ can be either positive or negative.

There is one element of the model that can at least partially characterize where x rises when cost uncertainty is increased and where x falls. It is the rate of profit of average costs $\Pi(W_0)$ relative to the riskless rate of return r_0. We next show that if $\Pi(W_0) > r_0$, then investment is negatively related to uncertainty, while if $\Pi(W_0) < r_0$, the relationship is positive. Consider first the case where $\Pi(W_0) > r_0$. Let ε_0 satisfy $\Pi(W_0+\varepsilon_0) = r_0$. The functions x_1 and x_2 defined above are shown in this case in fig. 1. As long as $x_1 > x_2 + 1$, we have the corner solution $x = 1$ and changes in ε have no effect on x. If at ε^* we move from the corner solution, the x_1 curve intersects the $x_2 + 1$ curve from above. Hence in the neighborhood of ε^* we have $\mathrm{d}x/\mathrm{d}\varepsilon < 0$.

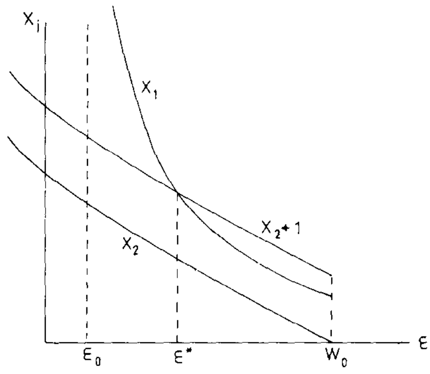

Fig. 1

Consider now the case where $\Pi(W_0) < r_0$. Let ε_1 satisfy $\Pi(W_0 - \varepsilon_1) = r_0$. The functions x_1 and x_2 in this case are shown in fig. 2. As long as ε is less than ε^{**} consumers do not hold capital in technology III and $x = 0$. At ε^{**} and beyond the economy leaves the corner solution and moves to $x > 0$. Notice that since x_2 intersects x_1 from above at ε^{**}, we have in the neighborhood of ε^{**}: $\mathrm{d}x/\mathrm{d}\varepsilon > 0$.

Hence, if $\Pi(W_0) > r_0$ cost uncertainty has a negative effect on investment, while if $\Pi(W_0) < r_0$ it has a positive effect, for relatively small ε. These results are also valid in the more general case, where the utility function is not necessarily logarithmic.

The same dividing line $\Pi(W_0) = \Pi[\mathrm{E}_{t-1}(W_t)] = r_0$ exists for the more general situations of richer distributions as well. This result can be explained by

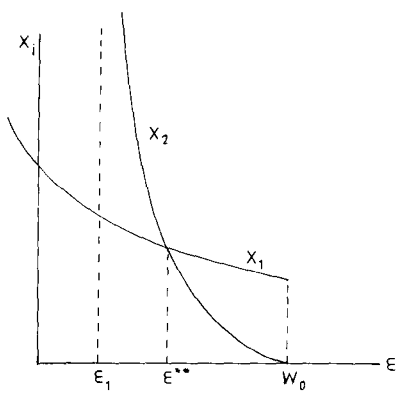

Fig. 2

the following heuristic argument. If $\Pi(W_0) > r_0$, then in the no uncertainty case capital of technology III is a superior asset and hence x is at a corner solution and is equal to 1. Now let us raise uncertainty, leaving average costs $W_0 = \mathrm{E}_{t-1}(W_t)$ unchanged. If at all levels of risk x remains in the corner solution, then x is unaffected by risk. But if at some point $x < 1$, then in the neighborhood of the corner solution increased cost uncertainty reduces x. Hence if $\Pi(W_0) > r_0$, near the uncertainty case x is either unaffected or it is negatively affected by cost uncertainty. A similar argument applies for the opposite case.

5. The profit and utility functions

As realized in section 4, $\mathrm{d}x/\mathrm{d}\varepsilon$ is positive when $\Pi(W_0) < r_0$ and negative when $\Pi(W_0) > r_0$. But in the case where $\Pi(W_0) = r_0$ this derivative can be either positive or negative, depending on the functions Π and u.

Let us consider first the influence of the profit function Π, by assuming as in section 4 that the utility function is logarithmic and that production is given by the Cobb–Douglas function:

$$F(K, L) = AK^{1-\beta}L^{\beta}, \tag{15}$$

where $0 < \beta < 1$. The resulting convex profit function is

$$\Pi(W) = (1-\beta)\beta^{\beta/(1-\beta)}A^{1/(1-\beta)}W^{-\beta/(1-\beta)}. \tag{16}$$

If $\beta = \frac{2}{3}$, the profit function is

$$\Pi(W) = B_0 W^{-2},$$

where $B_0 = r_0 W_0^2$, since we deal with the case of $\Pi(W_0) = r_0$. When profits are described by this function the optimal share of capital x is

$$x = \frac{1+r_0}{r_0}\,\frac{3W_0^2 - \varepsilon^2}{4W_0^2 - \varepsilon^2}. \tag{17}$$

Hence x is a decreasing function of ε, and an increase in cost uncertainty in this case leads to a reduction in capital, unlike the results of Hartman and Abel.

If instead we examine the same Cobb–Douglas production function but with $\beta = \frac{1}{3}$ we get a different result. The resulting profit function is

$$\Pi(W) = B_1 W^{-1/2},$$

where $B_1 = r_0 W_0^{1/2}$, since $\Pi(W_0) = r_0$. We can use eq. (13) to calculate the derivative $\mathrm{d}x/\mathrm{d}\varepsilon$ and to show that $\mathrm{d}x/\mathrm{d}\varepsilon > 0$, so that cost uncertainty increases capital accumulation in this case.

It can be shown that when $\beta = \frac{1}{2}$, x is independent of ε, and cost uncertainty has no effect on investment. Hence $\frac{1}{2}$ is the value which separates between the cases of positive and negative effects of uncertainty on investment.

Changes in the profit function therefore alter the reaction of investment to cost uncertainty. The more concave the production function is in L, as β becomes smaller, the stronger is the positive effect of uncertainty on investment, through the convex profit function.

Not only the profit function affects the sign of $\mathrm{d}x/\mathrm{d}\varepsilon$ in the case where $\Pi(W_0) = r_0$, but the utility function as well. The degree of relative risk aversion can also alter the effect of cost uncertainty on investment. In order to examine this issue consider the first-order condition (10) in the case of the two-point distribution (12):

$$\begin{aligned} MU(x,\varepsilon) &= \tfrac{1}{2}u'\{x[\Pi(W_0-\varepsilon)-r_0]+1+r_0\}[\Pi(W_0-\varepsilon)-r_0] \\ &\quad -\tfrac{1}{2}u'\{x[\Pi(W_0+\varepsilon)-r_0]+1+r_0\}[r_0-\Pi(W_0+\varepsilon)] \\ &= 0. \end{aligned} \tag{18}$$

Since MUx is always negative we get

$$\operatorname{sign}\left(\frac{\mathrm{d}x}{\mathrm{d}\varepsilon}\right) = \operatorname{sign} MU_\varepsilon. \tag{19}$$

But straightforward calculation shows that

$$MU_\varepsilon = (1-\alpha)\mathrm{E}\left(u'\frac{\mathrm{d}\Pi}{\mathrm{d}\varepsilon}\right) - (1+r_0)R\,\mathrm{E}\left(u''\frac{\mathrm{d}\Pi}{\mathrm{d}\varepsilon}\right), \tag{20}$$

where α is the measure of relative risk aversion. Eq. (20) shows how changes in the degree of risk aversion further affect the sign of MU_ε. Notice that

$$\mathrm{E}\left(u'\frac{\mathrm{d}\Pi}{\mathrm{d}\varepsilon}\right) > 0,$$

as can be seen from eq. (18) and from the fact that

$$\frac{\Pi(W_0-\varepsilon)-r_0}{-\Pi'(W_0-\varepsilon)}>\varepsilon, \qquad \frac{r_0-\Pi(W_0+\varepsilon)}{-\Pi'(W_0+\varepsilon)}<\varepsilon.$$

Notice that in the separating case of logarithmic utility $\alpha=1$, $MU_\varepsilon = -(1+r_0)\mathrm{E}(u''(\mathrm{d}\Pi/\mathrm{d}\varepsilon))$ and this term can be either positive or negative. If the measure of relative risk aversion α is higher than 1, then a negative term is added and MU_ε falls. If the measure of relative risk aversion is less than 1, then a positive term is added and MU_ε rises. It is therefore clear that the utility function affects the responsiveness of investment to uncertainty and it can be said that the more risk-averse consumers are, the more likely it is that increased cost uncertainty would reduce the rate of investment.

Thus far we have seen that if consumers are risk-averse, the effect of cost and price uncertainty on capital accumulation is no longer anywhere positive as with risk-neutral consumers. This effect can be positive or negative and it depends on a variety of characteristics of the economy: the riskless rate of return, the profit function, and the degree of risk aversion. These results were derived in the case of the rather simple two-point distribution. It can be shown though that this distribution is quite indicative as to the general effect of increased cost uncertainty on capital accumulation, and that the results hold for a wider family of distributions.[7]

6. Summary

The main argument of this paper is that the effect of cost and price uncertainty on investment should be analyzed in a framework that takes into consideration risk aversion of shareholders. Former analysis which have considered only the effect of such uncertainty on expected profitability came out with the strong result that the effect is positive. This paper shows that if consumers are risk-averse, this result no longer holds. Investment may increase or decrease as a result of increased uncertainty, and the direction of the effect depends on many variables: on the degree of risk aversion, on the profit function, on the riskless rate of return, etc. The analysis is of course more complicated when the two effects are taken into account, and the results are ambiguous. But I believe that this is a more appropriate way to analyze the issue. Notice that the effect of price uncertainty on investment is important for an analysis of various economic issues. Inflation usually raises price dispersion and price uncertainty. Hence the effect of inflation on capital accumulation depends, among other things, on the effect of price uncertainty on investment.

[7]See Zeira (1989).

Thus one possible extension of this line of research is to reexamine the issue of the effect of inflation on growth, through price uncertainty.

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