# RISK AND CAPITAL ACCUMULATION IN A SMALL OPEN ECONOMY*

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This paper examines the dynamics of capital accumulation in a small open economy where home capital is risky and consumers are risk-averse. It is assumed that the economy participates in perfect international bond markets but that risky home capital is held by domestic residents only. Under these assumptions the rate of investment is no longer independent of the saving rate, and they are positively related. As a result, a rise in savings does not increase foreign investment by the same amount but by less, and in some situations the quantity of foreign assets may even decrease.

## I. INTRODUCTION

This paper presents a simple general equilibrium model of a small open economy with risky capital and risk-averse consumers. The model is used to analyze the relationship between the rate of saving—private or public—and the accumulation of capital and foreign assets. In standard open economy models with perfect capital mobility and riskless capital, we find that investment is independent of domestic savings, and it depends only on technology and the world interest rate, due to perfect substitutability between home capital and foreign bonds. According to these models, an increase in savings shows up one for one in the current account and the holdings of foreign assets, as in Blanchard [1983], Persson [1983], and other studies. This conclusion stands in sharp contrast with the empirical findings of Feldstein and Horioka [1980], Feldstein [1983], Summers [1985], and others, that have found high positive correlations between saving and investment rates, both in OECD countries and in LDCs.

These empirical results led Feldstein and Horioka to the conclusion that capital mobility is yet far from being perfect. This conclusion is challenged both by Obstfeld [1985] and Summers [1985], who claim that there may be other explanations for the positive correlation between saving and investment. Obstfeld suggests that there may be a common cause that shifts both investment

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and saving in the same direction, while Summers suggests that governments counterreact to current account imbalances, thus leading to equalization of saving and investment. This paper considers instead the theoretical possibility that investment does indeed depend on saving, in the presence of risk and some restrictions on capital mobility.

In the model that is presented in this paper, capital accumulation is shown to be positively related to the rate of saving, under the two following major assumptions:

a. Returns from capital are subject to random shocks, and hence capital is a risky asset. Since consumers are risk-averse, the expected rate of return on capital is no longer equalized to the world rate of interest.

b. There is perfect international mobility of bonds, but capital is owned by domestic residents only. A similar distinction between short-term liquid assets which are mobile and long-term capital which is not is observed by Feldstein and Horioka [1980]. They give four reasons for the immobility of long-term capital: official restrictions on capital movements, institutional rigidities in domestic capital markets, international differences in tax rules and most importantly asymmetric information between countries on profitability, of capital: "For most investors, the uncertainties and risks associated with foreign investment are perceived as so great that investment is restricted to the domestic economy" [Feldstein and Horioka, 1980, p. 316]. It is this explanation that seems most appropriate to our model, where capital profitability fluctuates both due to random shocks and due to changes in capital stock. We assume that domestic residents have an informational advantage over foreigners with respect to their country's capital accumulation process. That makes domestic capital an asset with smaller variability to domestic residents than to foreigners, and that justifies the above assumption.[1] Another possible deterrent to direct ownership of foreign capital is the high risks involved with absentee ownership, especially with regard to housing and similar types of investment.

The outcome of these two assumptions is examined within a model that is kept as simple as possible and yet is capable of presenting both intertemporal saving and investment decisions and portfolio diversification decisions. It is a PPP model with one physical good and no money, but with two assets: domestic risky

1. In a recent empirical paper Cooper and Kaplanis [1985] estimate the barrier due to information-gathering costs on foreign investments in a sample of fourteen countries to be 4 to 6 percent of the value of net foreign investment.

immobile capital and riskless perfectly mobile bonds. It is an overlapping generations model, where expectations are assumed to be rational.[2] The model examines the dynamic path of capital and foreign assets accumulation and analyzes the effects of changes in private or in public saving. As mentioned above, investment is shown to be positively correlated with saving. Consequently, foreign assets accumulation increases by less than the increase in savings and in some cases, when foreign debt is high, foreign assets holdings may even decrease.

The paper is organized as follows. Section II presents the model, and Section III analyzes the dynamics of capital and foreign assets accumulation. In Section IV we examine the effects of changes in the rate of saving, while Section V summarizes the paper.

## II. The Model

There is only one physical good in the home country and in the rest of the world, that is used both for consumption and investment. The production of this good at the home country at time $t$ is

$$Y_t = F(K_t,L_t) = PK_t^{\alpha}L_t^{1-\alpha}, \tag{1}$$

where $K_t$ is the amount of capital held at the beginning of period $t$, $L_t$ is the amount of labor hired, $P > 0$, $1 > \alpha > 0$.

The capital stock that is used in production depreciates at an uncertain rate, and that is the source of capital risk.[3] Capital depreciation in period $t$ is

$$\theta_t \cdot K_t, \tag{2}$$

where $\theta_t$ are independent, identically distributed random variables. Since $\theta_t$ are necessarily bounded, denote $\underline{\theta} = \inf \theta_t \geq 0$, $\bar{\theta} = \sup \theta_t \leq 1$. The average rate of depreciation is $\delta = E(\theta_t)$ and $0 < \delta < 1$.

The capital stock for production in period $t + 1$, $K_{t+1}$, is determined by gross investment in period $t$, $I_t$:

$$K_{t+1} = K_t - \theta_t K_t + I_t.$$

2. Recent examples for the use of overlapping generations models in the theory of open economies are Buiter [1981], Dornbusch [1985], Persson [1983], and Persson and Svensson [1985].

3. This specification of capital risk significantly simplifies the model. In a recent article Bulow and Summers [1984] use the same specification for capital risk and also provide some empirical evidence for the high variability of capital depreciation rates.

We assume that changes in capital are costly and that the adjustment costs are

$$K_t \cdot \frac{\gamma}{2} \cdot \left(\frac{K_{t+1} - K_t}{K_t}\right)^2, \tag{3}$$

where $\gamma > 0$. This specification of constant returns to scale adjustment costs is quite standard except that here the replacement for depreciated capital is costless, and only net changes in capital are costly. Adjustment costs that depend on net investment can be viewed as training and organization costs.[4]

Equities of firms traded in period $t$ entitle their owners to profits from period $t + 1$ on. Hence the value of capital in the stock market in period $t$ $V_t$ depends on $K_{t+1}$, the amount of capital after investment and depreciation. Since technology is of constant returns to scale in $K$, $L$, and $I$, the value of capital is proportional to $K_{t+1}$, due to arbitrage considerations, and there is a price of capital $q_t$ such that

$$V_t = q_t \cdot K_{t+1}.$$

The firm maximizes its net value $NV_t$:

$$NV_t = \max_{L_t, K_{t+1}} \left[ q_t K_{t+1} + F(K_t, L_t) - W_t L_t - I_t - K_t \cdot \frac{\gamma}{2} \cdot \left(\frac{K_{t+1} - K_t}{K_t}\right)^2 \right], \tag{4}$$

where $W_t$ is the real wage at time $t$. Notice that $NV_t$ is the value of the receipts available to the owners of the firm in period $t$. The two first-order conditions are

$$F_L(K_t, L_t) = W_t,$$

and

$$K_{t+1} - K_t = K_t \cdot f(q_t), \tag{5}$$

where $f$ is defined by $f(q) = (q - 1)/\gamma$. The optimal net value of the firm is $NV_t = K_t[R(q_t) + F_K - \theta_t]$, where the function $R$ is defined by

$$R(q) = q + qf(q) - f(q) - (\gamma/2)[f(q)]^2.$$

4. If adjustment costs would depend on gross investment instead of net investment, the main analysis of the paper would remain the same, except that prices would be stochastic and instead of convergence to the steady state we would have convergence to a neighborhood of the steady state.

As for consumers in this model, they live for two periods, working and saving in the first period and dissaving in the second. We assume that there is no population growth and that all consumers are similar—hence the number of members in each generation can be normalized to one. In period $t$ the young generation supplies one unit of labor, earns $W_t$, consumes $C_{1,t}$, and saves $S_t = W_t - C_{1,t}$. In period $t + 1$ this generation, then old, will consume $C_{2,t+1}$, a random variable. In period $t$ the young generation maximizes

$$E_t(\log C_{1,t} + \beta \log C_{2,t+1}), \tag{6}$$

where $\beta > 0$.

Since labor supply is fixed and equal to one, the labor market equilibrium condition is

$$W_t = F_L(K_t,1) = (1 - \alpha)PK_t^{\alpha}. \tag{7}$$

The young allocate their savings between capital and bonds: $S_t = q_t K_{t+1}^d + B_t$, where $B_t$ can be either positive or negative, in case of lending or borrowing. Let $r$ be the world rate of interest, and let $x_t$ be the portfolio share of capital: $q_t K_{t+1}^d = x_t S_t$ and $B_t = (1 - x_t)S_t$.

The utility maximization of the young is therefore

$$\max_{S_t,x_t} E_t \left\{ \log(W_t - S_t) + \beta \log S_t + \beta \log\left[(1 + r)(1 - x_t) + x_t \frac{R(q_{t+1}) + F_K(K_{t+1},1) - \theta_{t+1}}{qt}\right]\right\}, \tag{8}$$

subject to

$$S_t \geq 0, \quad x_t \geq 0.$$

The optimal values of $S_t$ and $x_t$ determine the quantity demanded of capital by the young and that determines the capital market equilibrium condition:

$$S_t \cdot x_t = K_t \cdot q_t \cdot [1 + f(q_t)], \tag{9}$$

where we rely on the assumption that home capital is held by domestic residents only.

As is clear from (8), neither $S_t$ nor $x_t$ depends on $\theta_t$; and hence $q_t$, which is determined by (9), does not depend on $\theta_t$, and is a nonstochastic variable. Since the same argument holds for all future periods as well, $q_{t+1}$ is nonstochastic too, and due to the rational expectations assumption it is known with certainty. We can therefore derive from equations (8) and (9) an explicit functional relationship between the price of capital $q_t$, next period price $q_{t+1}$ and $K_t$. It can be shown that the optimal solution to (8) is given by

$$S_t = \frac{\beta}{1+\beta} W_t = \frac{\beta}{1+\beta}(1-\alpha)PK_t^{\alpha}, \tag{10}$$

and by

$$x_t = \inf\left\{x: x \geq 0,\quad E_t\left[\frac{R(q_{t+1}) + \rho_{t+1} - \theta_{t+1}}{q_t} - 1 - r\middle/ 1 + r + x\left(\frac{R(q_{t+1}) + \rho_{t+1} - \theta_{t+1}}{q_t} - 1 - r\right)\right] < 0\right\}, \tag{11}$$

where $\rho_{t+1}$ denotes $F_K(K_{t+1},1)$. Notice that at the share $x_t$ there is no default risk and all debt is fully paid back.

The value of $x_t$ as determined in (11) defines the optimal capital share function $x$: $x_t = x(q_t, q_{t+1}, \rho_{t+1}, r)$. If we impose an additional restriction on $\theta_t$, that for the relevant values of $\rho_t$ in this model we have: $\bar{\theta} \leq \rho_t$, namely, that the production profits minus depreciation costs are nonnegative, we can easily prove the following lemma.

LEMMA 1. The function $x$ satisfies $\partial x/\partial q_t \leq 0$, $\partial x/\partial q_{t+1} \geq 0$, $\partial x/\partial \rho_{t+1} \geq 0$, $\partial x/\partial r \leq 0$; and whenever $x > 0$, the inequalities are strict.

From equations (9), (10), and (11) we derive the explicit form of the capital market equilibrium condition:

$$(\beta/(1+\beta))(1-\alpha)PK_t^{\alpha-1}x\{q_t,q_{t+1},\alpha PK_t^{\alpha-1}[1+f(q_t)]^{\alpha-1},r\} - q_t[1+f(q_t)] = 0. \tag{12}$$

In the case of certainty, when $\theta_t \equiv \delta$, the two assets are perfect substitutes, and the rates of return are equalized:

$$\frac{R(q_{t+1}) + \rho_{t+1} - \delta}{q_t} = 1 + r. \tag{13}$$

## III. The Dynamics of Capital and Foreign Assets Accumulation

We now turn to describe the dynamics of the open economy, using the capital accumulation equation (5) and the capital market equilibrium condition (12).

A steady state is fully determined by the condition, $q_{t+1} = q_t = 1$. Hence the amount of capital at the steady state $K_\infty$ is uniquely determined by

$$(14) \qquad (\beta/(1 + (\beta))(1 - \alpha)PK_\infty^{\alpha-1}x(1,1,\alpha PK_\infty^{\alpha-1},r) - 1 = 0.$$

Notice that if capital is riskless, the steady state amount of capital $\bar{K}$ is determined by the condition, $F_K(\bar{K},1) - \delta = r$, and depends on $r$ and the technology alone. When capital is risky, $K_\infty$ depends on the propensity to save: $s = \beta/(1 + \beta)$ as well; and the expected rate of return on capital is higher than $r$: $F_K(K_\infty,1) - \delta > r$. Hence the steady state amount of capital is smaller than in the riskless capital case.

Notice that $K_\infty$ is bounded from below by $\underline{K}$, where $\underline{K}$ is defined by $F_K(\underline{K},1) - r - \bar{\theta} = 0$. As $K_\infty$ approaches $\underline{K}$, the share of capital $x$ tends to infinity. We therefore have $\underline{K} < K_\infty < \bar{K}$.

We now turn to the dynamics of the economy that are diagrammatically presented on the $(K_t,q_t)$ plane in Figure I. Capital increases when $q_t > 1$ and decreases when $q_t < 1$. The line $q_{t+1} = q_t$ describes capital market equilibrium situations where the price of capital stays unchanged.

PROPOSITION 1. If the coefficient of adjustment costs $\gamma$ is high enough, there exists a locally stable saddlepoint solution, $q_t = \phi(K_t)$, which leads the economy to the steady state, where $\phi(K_\infty) = 1$. Furthermore, $\phi$ is a decreasing function.

The proof is left to the Appendix.

The rationale to the condition in Proposition 1 is as follows: if the costs of adjustment are too low, the change in capital is so large that the economy experiences increasing fluctuations between the $q > 1$ and the $q < 1$ regions. This condition is due only to the discrete time structure of the model and is common to all similar adjustment costs models. Let us further assume that $\gamma$ is high enough to ensure that the convergence of the economy to the steady state is monotonic.

The saddlepoint solution, that is derived in Proposition 1, is described in Figure I as the $JJ$ curve along which the economy converges to the steady state $Q$, through accumulation or depreciation of capital. Let us now examine the current account $CA_t$ and the holdings of foreign assets $B_t$ along this path.

The current account is equal to home production plus interest payments minus consumption by the young, consumption by the old, investment, and adjustment costs. Explicit calculation of the

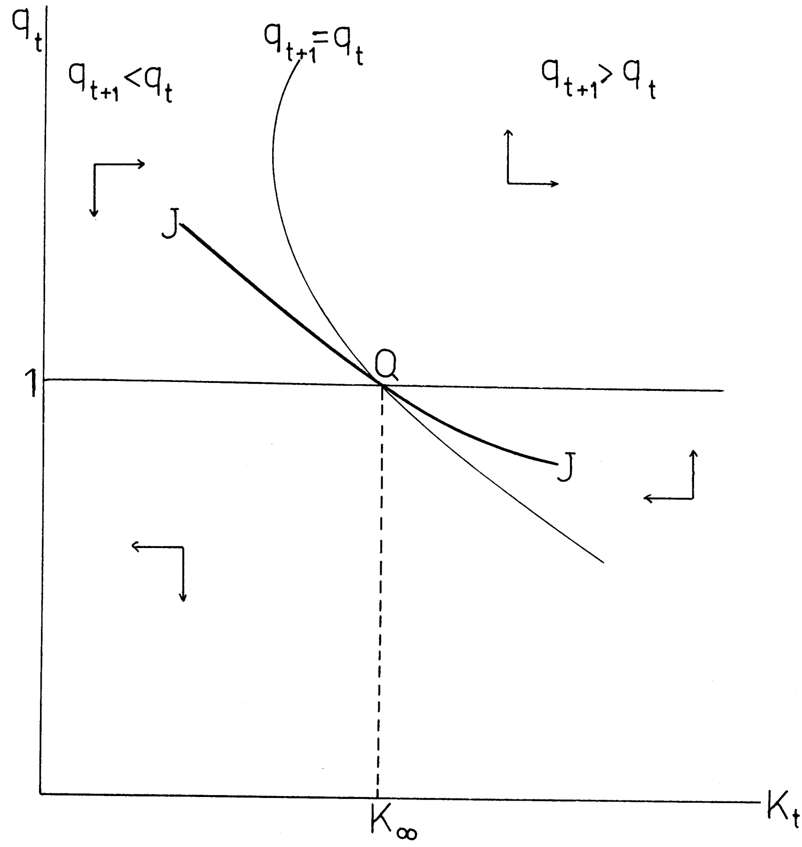


FIGURE I

current account yields

(15) $$CA_t = B_t - B_{t-1},$$

so that the rate of accumulation of foreign assets is equal to the current account. The amount of foreign assets held in the economy in period $t$, along the $JJ$ path, is

(16) $$B_t = sF_L(K_t,1) - K_t\phi(K_t)\{1 + f[\phi(K_t)]\}.$$

The quantity of capital $K_t$ is therefore the single dynamic variable that determines $B_t$ in each period. If $B_t$ is an increasing function of $K_t$, then when capital is accumulated, the economy runs a current account surplus; while if $B_t$ is decreasing, the economy runs a current account deficit as capital is accumulated. In both cases the current account is balanced at the steady state. This model therefore focuses on the capital accumulation process as a source of current account imbalances, as in Fischer and Frenkel [1972, 1974] and Frenkel [1971].

In order to see whether the economy runs a current account surplus or deficit, when capital is accumulated along the saddlepath, we examine the derivative of (16) at the steady state:

$$\frac{\partial B}{\partial K} = sF_{LK} - 1 - K_\infty \frac{\gamma + 1}{\gamma} \phi'(K_\infty), \tag{17}$$

which has an ambiguous sign. Capital accumulation can be accompanied either by current account surpluses or by current account deficits, as we see in Proposition 2, that examines this issue in the case of riskless capital.

PROPOSITION 2. If capital is riskless, $\partial B/\partial K$ can be either positive or negative. It is negative when $r$ and $\delta$ are low, and positive when they are high.

The proof is left to the Appendix.

The analysis of the risky capital case is more complicated than the riskless case, but the ambiguity remains.

## IV. The Effect of Increased Savings

The saddlepath the economy follows is dependent on various parameters of the economy: the propensity to save $s$, the world interest rate $r$, the productivity coefficient $P$, and the distribution of the depreciation rate $\theta$. The changes in the dynamic path of the economy can be traced by examining the changes in the steady state.

A rise in the propensity to save $s$, through a rise in the rate of time preference $\beta$, increases the steady state quantity of capital $K_\infty$ as can be seen from equation (14). The $JJ$ curve therefore shifts upward; the economy jumps to the new saddlepath; $q$ rises; and investment increases. Hence, if individuals save more of their income, the rate of capital accumulation is higher.

Notice that investment increases not only in the short run but in the long run as well. The average rate of investment in a steady state is $E(I/Y) = (\delta/P)K_\infty^{1-\alpha}$, and an increase in the rate of saving leads to a higher $K_\infty$.[5]

5. This result too is in accordance with the findings of Feldstein and Horioka [1980] and Feldstein [1983].

It is important to note that this result—that investment is positively correlated with savings—depends crucially on both assumptions of uncertainty and portfolio limitations. Notice that in the riskless capital case, when $\theta \equiv \delta$, a change in saving has no effect on capital accumulation, since the long-run amount of capital $\bar{K}$ is independent of $s$: $F_K(\bar{K},1) = r + \delta$. Hence if capital is riskless, investment is independent of saving, as long as bonds are perfectly mobile, as in Blanchard [1983] and Persson [1983]. Capital risk alone is also not sufficient, since if capital is risky but equity capital is perfectly mobile, then equation (9) holds in the world market for risky capital instead of the domestic market. Hence changes in saving in a small economy have a negligible effect on the world's saving and hence no effect on capital accumulation. Thus, the two assumptions together are necessary for the main result to hold.

In order to derive some quantitative estimate for the effect of saving on investment in this model, consider the following example of a distribution function: $\theta$ is $\delta + \epsilon$ with probability ½ and $\delta - \epsilon$ with probability ½. It can be shown that under this distribution the average long-run rate of investment $E(I/Y)$ is determined by

$$[(r + \delta)^2 - \epsilon^2][E(I/Y)]^2 - [2\delta\alpha(r + \delta) + s\delta(1 - \alpha)(1 + r) \times (r + \delta)]E(I/Y) + \delta^2\alpha^2 + s(1 - \alpha)(1 + r)\delta^2\alpha = 0.$$

For a numerical valuation let us choose the following set of parameter values: $r = ½$, $\delta = ⅔$, since the depreciation rate usually exceeds the interest rate, $\epsilon = ⅓$, which is a medium degree of uncertainty, where the standard error is half the size of the mean, and the share of capital $\alpha$ is ⅓. Substituting these parameters, we get that for $s = ⅕$:

$$\frac{\partial E(I/Y)}{\partial s} = 0.108,$$

while for $s = ⅓$ the result of 0.08.

Notice that this is the long-run effect of a change in $s$, while the short-run effect is much stronger, since investment on the transition path is greater than at the steady state.

Another difference between this model and that of riskless capital is that the effect on foreign assets holdings is no longer unambiguously positive, since the rise in $s$ raises savings but increases the quantity of capital as well. Let us first examine the effect of an increase in $s$ on the long-run holdings of foreign assets, at the steady state:

(18) $$B_\infty = sF_L(K_\infty,1) - K_\infty.$$

Notice that the rise in $s$ increases the saving rate and the long-run real wage on the one hand and increases the stock of capital on the other hand. In Proposition 3 we prove that saving can sometimes have a negative effect on the long-run stock of foreign assets, when the economy is borrowing.

PROPOSITION 3. If the economy is a net creditor, $\partial B_\infty/\partial s$ is positive. If the economy is a net debtor, $\partial B_\infty/\partial s$ can be negative for low $s$.

*Proof.* By differentiating (18) and deriving $\partial K_\infty/\partial s$ from equation (14), we get $\partial B_\infty/\partial s < 0$ if and only if $x - 1 > (K_\infty/x)(-\partial x/\partial K_\infty) > 0$. Hence, when $x < 1$ and the economy is a net creditor, $B_\infty$ is positively related to $s$.

To demonstrate the possibility of a negative $\partial B_\infty/\partial s$, we look at the former example of a distribution function: $\theta$ is $\delta + \epsilon$ with probability ½ and $\delta - \epsilon$ with probability ½.

It can be shown that under this distribution $x = (1 + r)a/(\epsilon^2 - a^2)$, where $a = PK_\infty^{\alpha-1} - r - \delta$, and that

$$-\frac{K_\infty}{x^2}\frac{\partial x}{\partial K_\infty} = (1 - \alpha)\frac{a + r + \delta}{a^2}(\epsilon + a)^2\left[1 + \left(\frac{a - \epsilon}{a + \epsilon}\right)^2\right].$$

As $s$ becomes smaller, $K_\infty$ approaches $\underline{K}$; $x$ tends to infinity; and $a$ tends to $\epsilon$. Hence for small enough $s$ the condition $x - 1 > (-K_\infty/x) \times (\partial x/\partial K_\infty)$ is met if

$$4(1 - \alpha)(\epsilon + r + \delta) < 1. \qquad \text{Q.E.D.}$$

The short-run effect of a rise in $s$ on the accumulation of foreign assets is that the increase in saving does not show up one for one in the current account, since decreased consumption by the young is accompanied by increased investment and increased consumption by the old, due to a higher $q_t$. There are even situations where a rise in $s$ creates a current account deficit if, for example, $\partial B_\infty/\partial s$ is negative and $\partial B_t/\partial K_t$ is positive. It can be shown though, that if the economy is lending, a rise in saving always creates a current account surplus, of a smaller size.[6]

In a similar way the model enables us to analyze the effects of changes in the other parameters: the world rate of interest $r$, the

6. The proof of this proposition is similar to Zeira [1985].

coefficient of productivity $P$, and the distribution of the depreciation rate $\theta$.[7]

The model presented in this paper can be extended to include a government with public consumption, a lump sum tax and a public debt, which is tradable in the world bonds market. The dynamic analysis is similar, provided that taxes are not too high and that the budget is balanced in the long run. The results derived in former sections can be extended to an economy with government as well. Thus, a fiscal expansion raises future expected taxes, lowers savings, and hence crowds out investment and reduces $K_\infty$. As a result, the amount of foreign assets is reduced by less than in the riskless capital case and may even increase, under conditions similar to those of Proposition 3.[8]

## V. Summary

In this paper a general equilibrium intertemporal model of a small open economy is developed where uncertainty is explicitly taken into consideration by consumers. The major assumptions that characterize the model are that capital is risky and is a nontraded asset, while bonds are perfectly internationally mobile. This model is used to examine the issue of the effect of changes in saving on the accumulation of capital and of foreign assets.

The major result of the model is that an increase in private or public savings leads to a rise in the price of capital, to a higher rate of investment, and to a larger long-run quantity of capital. As a result, the current account and the quantity of foreign assets increase by less than savings. There is even a possibility of a decrease in the amount of foreign assets held.

In this model capital is a risky asset, but firms and individuals can still lend or borrow any quantity in the world bonds market. Hence, the only effect of risk in this model is through portfolio decisions of consumers. But borrowing for risky investments is

7. Increased uncertainty of $\theta$ decreases the demand for capital and hence lowers the rate of investment and creates an immediate current account surplus. Notice that this result differs from Hartman [1972] and Abel [1984], since they do not take into account risk-averse consumers.

8. In this model a tax cut increases the current account deficit by more than a budget deficit of the same size that is caused by increased government expenditures. Two other results of the model are that a temporary fiscal expansion increases the current account deficit by more than a permanent one, and that a future expected expansion creates a current account surplus at the present. These two results are similar to Sachs [1982], though in his paper they are due to consumption smoothing and here they are due to intertemporal portfolio considerations.

usually limited due to default risk and credit rationing. It therefore seems to me that an important step forward in developing a general equilibrium macroeconomic theory of the open economy would be to incorporate in it not only capital risk, but also default risk and some resulting forms of credit rationing and international financial intermediaries. That can lead us to a better understanding of the issues of foreign debt and international capital movements.

## APPENDIX

### *1. Proof of Proposition 1*

The dynamic equations of the model are the capital accumulation equation: $K_{t+1} = K_t + K_t(q_t - 1)/\gamma$, and the relationship between $q_{t+1}$, $q_t$, and $K_t$, that is derived from the capital market equilibrium condition (12). In order to examine the local stability of the system, we have to examine

$$A = \begin{bmatrix} \dfrac{\partial K_{t+1}}{\partial K_t} & \dfrac{\partial K_{t+1}}{\partial q_t} \\ \dfrac{\partial q_{t+1}}{\partial K_t} & \dfrac{\partial q_{t+1}}{\partial q_t} \end{bmatrix} \tag{A.1}$$

with the derivatives calculated at the steady state $(K_\infty,1)$. All elements of $A$ are positive, $a_{11} = 1$ and $a_{22} > 1$, since $-\partial x/\partial q_t > \partial x/\partial q_{t+1}$ at the steady state. Hence, if $\lambda_1$ and $\lambda_2$ are the two characteristic values of $A$, then $\lambda_2 > 1$, and $\lambda_1 < 1$. It can be shown that $\lambda_1 > -1$ if and only if

$$a_{21} \cdot (K_\infty/\gamma) < 4 + 2(a_{22} - 1), \tag{A.2}$$

and that is a necessary and sufficient condition for the existence of a stable saddlepoint solution. Hence $a_{21}\ K_\infty/\gamma < 4$ is a sufficient condition for local stability and as $a_{21}$ depends on $K_\infty$ only (not on $\gamma$), this condition is met for all relevant $K_\infty$ for a high enough $\gamma$.

The slope of the saddlepoint path, according to Blanchard and Kahn [1980], is given by

$$\phi'(K_\infty) = (\lambda_1 - a_{11})/a_{12} < 0. \tag{A.3}$$

Hence $\phi$ is a decreasing function. Q.E.D.

*2. Proof of Proposition 2*

If capital is riskless, it follows from (13) that

$$\text{(A.4)} \qquad \begin{aligned} a_{11} &= 1, \quad a_{12} = K_\infty/\gamma, \\ a_{21} &= \alpha(1-\alpha)PK_\infty^{\alpha-2}, \\ a_{22} &= 1 + r + 1/\gamma\, \alpha(1-\alpha)PK_\infty^{\alpha-1}. \end{aligned}$$

Notice that $\alpha PK_\infty^{\alpha-1} = r + \delta$. Therefore,

$$\text{(A.5)} \qquad \begin{aligned} \frac{\partial B}{\partial K} &= sF_{LK}(K_\infty,1) - 1 - K_\infty \frac{\gamma+1}{\gamma}\phi'(K_\infty) \\ &= s(1-\alpha)(r+\delta) - 1 + \frac{\gamma+1}{2} \\ &\quad \times \left\{ \sqrt{\left[r + \frac{1-\alpha}{\gamma}(r+\delta)\right]^2 + 4\frac{1-\alpha}{\gamma}(r+\delta)} \right. \\ &\quad \left. - r - \frac{1-\alpha}{\gamma}(r+\delta) \right\}. \end{aligned}$$

From equation (A.5) we learn that when $r$ and $\delta$ are low, $\partial B/\partial K$ is negative. If $r + \delta$ is high enough, $\partial B/\partial K$ is positive.

Q.E.D.

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