Investment as a Process of Search

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This paper analyzes optimal capital accumulation in the face of "structural uncertainty," in which the firm does not fully know its own profit function and can discover it only through further investment. It is shown that under structural uncertainty capital accumulation is gradual even when adjustment costs are linear and not convex. It is further shown that structural uncertainty creates an incentive to market research. The paper also presents an example of structural uncertainty in which an additional price uncertainty has a negative effect on investment, contrary to the standard models.

I. Introduction

Gradual accumulation of capital by firms is usually explained either by convex adjustments costs, as in Lucas (1967), Gould (1968), and Treadway (1969), or by financial constraints induced by imperfect capital markets, as in Steigum (1983). This paper offers a third line of explanation for the phenomenon of gradual capital adjustment, as the firm's optimal response to "structural uncertainty."

Structural uncertainty is the uncertainty that arises when the firm does not fully know its own profit function, which relates profits to capital stock. The exact levels of profits at larger amounts of capital can be discovered only by actually increasing the quantity of capital, that is, by "getting there." Investment under structural uncertainty therefore creates an interaction between the accumulation of capital

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and the accumulation of information. Such interaction does not exist under the standard types of uncertainty, as in Lucas and Prescott (1971) or Bernanke (1983).

In the face of structural uncertainty, investment becomes a process of learning the profit function and of search for the optimal amount of capital. The main result of the paper is that if there are adjustment costs to investment, this search becomes gradual since the firm slows down in order to lower the risk of overinvestment. The important point is that, for this result to hold, adjustment costs no longer need to be convex, and it is sufficient that they be linear.[1]

It is also shown in the paper that investment under structural uncertainty and linear adjustment costs is positively related to the rate of profit and negatively related to the rate of interest. These results are similar to those of the standard model. The structural uncertainty framework is also applied to the analysis of two additional issues: "market research" and the effect of price uncertainty on investment. Unlike the standard model, in which price uncertainty tends to increase the rate of investment, as in Hartman (1972) and Abel (1984*b*), under structural uncertainty it may lead to a lower rate of investment.

The paper is organized as follows. Section II presents the basic model, while Section III discusses the optimal investment function by use of two examples. Section IV extends the analysis to the issues of market research and price uncertainty, and Section V summarizes the paper.

II. The Model

Consider a competitive firm that produces a single good. The quantity produced in period t is $\min(K_t, K^*)$, where K_t is the stock of the capital at time t and K^* is the upper bound to productive capital. The exact value of K^* is unknown as long as $K_t \leq K^*$ and is discovered only when $K_t > K^*$, as the firm observes the quantity produced. All the a priori information on K^* is summarized in a subjective probability distribution given by a density function $f(K^*)$ for $K^* > 0$. If K^* is not approached yet at time t, the distribution is updated to

$$f(K^*, K_t) = \frac{f(K^*)}{\int_{K_t}^{\infty} f(x)dx} \tag{1}$$

for $K^* \geq K_t$, and 0 elsewhere.

[1] Notice in comparison that, under the standard type of uncertainty, convexity of adjustment cost is a necessary condition for gradual capital adjustment, as can be seen in Lucas and Prescott (1971).

The price p of the good the firm produces is determined in the competitive market.[2] Production costs are cK, with $c > 0$, and I assume that the scale of production can be immediately reduced once the firm passes K^* in order to minimize production costs. Operating profits in period t are therefore equal to $\alpha \cdot \min(K_t, K^*)$, where $\alpha = p - c$.

I further assume that the price of an investment good is P_k while the resale value of an already installed investment good unit is zero. This is equivalent to the assumption of linear adjustment costs. Capital is assumed to be nondepreciating.

The firm maximizes the expected discounted sum of present and future cash flows:

$$\max E_0 \sum_{t=0}^{\infty} \frac{1}{(1 + r)^t} \{\alpha \min(K_t, K^*) - \max[P_k(K_{t+1} - K_t), 0]\}, \quad (2)$$

where r is the real rate of interest and the distribution of K^* follows equation (1).

I add the assumption

$$\frac{\alpha}{r} > P_k, \quad (3)$$

which implies that the cost of an investment unit is less than the discounted sum of the future profits it generates if K^* is not reached.

III. Optimal Investment under Structural Uncertainty

The optimal amount of capital is K^*, which is unknown in advance. For the analysis of optimal capital accumulation until K^* is discovered, let us adopt a slight change in notation and denote by K_t the amount of capital planned for period t if K^* is yet unknown in period $t - 1$. All K_t, $t \geq 1$, are known with certainty at time 0 since all future probabilities can be calculated in advance by (1). It can therefore be shown that (2) is equivalent to the following optimization:

$$\max_{\{K_t\}_{t\geq 1}} \left\{ \sum_{t=0}^{\infty} \frac{1}{(1 + r)^t} \frac{\alpha}{r} \int_{K_t}^{K_{t+1}} K^* f(K^*) dK^* + \sum_{t=0}^{\infty} \frac{1}{(1 + r)^t} [\alpha K_t - P_k(K_{t+1} - K_t)] \int_{K_t}^{\infty} f(K^*) dK^* \right\}. \quad (4)$$

[2] Structural uncertainty is modeled in this section as uncertainty of the production function. In Sec. IV an alternative model is presented in which structural uncertainty is related to the demand function.

The first-order condition for (4), with respect to K_{t+1}, is

$$-\int_{K_t}^{\infty} f(K^*)dK^* + \frac{1+\rho}{1+r}\int_{K_{t+1}}^{\infty} f(K^*)dK^* + \frac{1}{1+r}(K_{t+2} - K_{t+1})f(K_{t+1}) = 0, \tag{5}$$

where $\rho = \alpha/P_k$ is the rate of profit, which is greater than r. It follows from (5) that optimal investment depends on ρ, on r, and on the distribution of K^*.

Consider first the case in which K^* is bounded by $B < \infty$. The firm never jumps to B since otherwise, if $t + 1$ is the first period at B, we get $K_t < K_{t+1} = K_{t+2} = B$, and that contradicts (5). Hence capital is accumulated gradually, $K_{t+1} > K_t$ for all t, and

$$K_t \underset{t\to\infty}{\longrightarrow} B. \tag{6}$$

In order to gain a better insight into the dynamics of investment, let us consider the example of the uniform distribution: $f(K^*) = 1/B$ for all $0 \leqslant K^* \leqslant B$, and 0 elsewhere. This distribution describes the situation in which the only a priori information on K^* is that it is bounded by B. From conditions (5) and (6) we get that, in this example, investment at time t is

$$I_t = K_{t+1} - K_t = \Psi \cdot (B - K_t), \tag{7}$$

where Ψ is the positive solution to

$$\Psi^2 + \rho\Psi - (\rho - r) = 0. \tag{8}$$

Investment in this example is therefore a constant percentage of the difference between the upper bound and the current capital stock. Notice that $\Psi = \Psi(\rho, r)$, with $0 < \Psi < 1$ and $\Psi_1 > 0$, $\Psi_2 < 0$. Hence investment is positively related to profitability and negatively related to the rate of interest and to the price of the investment good. Note that, as ρ and r become closer to each other, Ψ approaches zero, which implies that capital accumulation not only is gradual but also can be rather slow. The average time length of capital accumulation in this example is $\Psi + 2\Psi(1 - \Psi) + 3\Psi(1 - \Psi)^2 + \cdots = 1/\Psi$, which is larger than one period. The main results that are derived from this example can be extended to the more general case of bounded K^*.[3]

[3] It can be shown that if K^* is bounded and if the optimal solution to (4) is unique, investment is positively related to ρ and negatively related to r. These results are quite general since the assumption that K^* is bounded is quite realistic. It can also be shown that for a wide family of distribution functions it holds that investment tends to zero as ρ approaches r.

Let us now consider as a second example the exponential distribution, in which K^* is unbounded. The initial density function is $f(K^*) = \lambda e^{-\lambda K^*}$, for all $K^* \geq 0$, with $\lambda > 0$. Since the conditional probability at a stock of capital K satisfies $f(K^*, K) = f(K^* - K)$, the firm's optimal investment is independent of the amount of capital it holds. Hence it follows from equation (5) that optimal investment in each period is

$$I = \phi \cdot \frac{1}{\lambda}, \tag{9}$$

where ϕ is the positive solution to

$$(1 + r)e^{\phi} - \phi - (1 + \rho) = 0. \tag{10}$$

Similarly to the former example, we have $\phi = \phi(\rho, r)$, $\phi_1 > 0$, $\phi_2 < 0$, and ϕ approaches zero as r and ρ become closer. The average time length of capital accumulation in this example is $1/(1 - e^{-\phi})$, which similarly to the former example is more than one period and can even be quite lengthy if r is close to ρ.

IV. Extensions

Under structural uncertainty, information on the profit function can be acquired either by increasing K, as discussed in former sections, or by conducting research. This can be either market research on demand or technological research on production possibilities. In both cases the research aims at narrowing the probability distribution of K^*. In order to examine the effect of such research on the value of the firm, consider a simple example of a uniform distribution: $f_D(K^*) = 1/D$ for $A - (D/2) \leq K^* \leq A + (D/2)$ and 0 elsewhere, where A is average K^* and $D > 0$. The optimal value of the firm $V_D(K_0)$ for $K_0 < A - (D/2)$ and its derivative can be calculated to get

$$-\frac{\partial V_D}{\partial D} = \frac{\sqrt{\alpha^2 + 4\alpha P_k - 4rP_k^2}}{4} - \frac{\alpha}{4} > 0. \tag{11}$$

Hence market research, which is ex ante viewed as decreasing the uncertainty of K^*, raises the value of the firm and can be conducted even if it is costly. The intuitive explanation for this is that market research reduces costly overinvestment. It also follows from (11) that the optimal amount of market research depends positively on α, negatively on r, and ambiguously on P_k. Other examples provide similar results.

Another issue that structural uncertainty can shed new light on is the effect of price uncertainty on investment. From a series of papers by Hartman (1972), Pindyck (1982), and Abel (1983, 1984*a*, 1984*b*) emerges the conclusion that in the standard model price uncertainty raises or leaves unchanged the rate of investment. Alternatively, consider a firm with an unknown demand schedule that is searching for the optimal capital stock. For such a firm price uncertainty obstructs search and may slow capital accumulation. This possibility is demonstrated by the following simple example.

Consider a monopolistic firm that can charge a price x_t for its good at time t, which is given by

$$x_t = \min\left(P_t, \frac{P_t Q^*}{Q_t}\right), \tag{12}$$

where P_t is the general price level, Q_t is the quantity produced by the firm, and Q^* is unknown, with a probability density function $f(Q^*)$. Assume further that $\{P_t\}$ are independent identically distributed unbounded random variables. Price level P_t is an aggregate variable that is revealed to the public only after some time. It is unknown during period t, when investment takes place. It becomes known only at the end of period t, and only then does the firm discover whether Q^* has been reached in period t or not. The rest of the model is similar to the former model: $Q_t = K_t$, production and investment costs are the same, p is the expectation of P_t, and we assume that $\alpha = p - c > rP_k$.

Let V_t be the optimal value of the firm when Q^* is unknown; then $V_t = V(K_t, K_{t-1})$. It can be shown that the value function V satisfies the following Bellman condition:

$$V(K_0, K_{-1}) = \max_{I \geq 0} \left\{ -cK_0 - P_k I + \left(p + \frac{\alpha}{r}\right) \int_{K_{-1}}^{K_0} K^* f(K^*, K_{-1}) dK^* \right. \tag{13}$$
$$\left. + \left[pK_0 + \frac{1}{1+r} V(K_0 + I, K_0)\right] \int_{K_0}^{\infty} f(K^*, K_{-1}) dK^* \right\}.$$

Solving (13) for the specific exponential distribution $\lambda e^{-\lambda Q^*}$ by assuming that the firm has acted optimally in former periods as well leads to an optimal rate of investment, which is independent of K_0, $I = \theta/\lambda$, where

$$(1 + r)e^{\theta} - \theta e^{-\theta} \frac{1 + (c/P_k)}{1 + r} - 1 - \frac{pe^{-\theta} - c}{P_k} = 0. \tag{14}$$

A comparison of equations (14) and (10) shows that $\theta < \phi$. Hence investment with price uncertainty is smaller than without it.

V. Summary

This paper focuses on the importance of structural uncertainty to the dynamics of capital accumulation. It is shown that structural uncertainty can serve as an explanation for gradual capital accumulation when adjustment costs are linear and not convex, as an incentive to market research, and it can also shed new light on the effect of price uncertainty on investment. Structural uncertainty therefore has significant effects on investment, both quantitatively and qualitatively.

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